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The Analysis of Tidal Observations

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VI. *The Analysis of Tidal Observations.*

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1. *Introduction.*

The Tidal Institute was founded in the year 1919 and researches on tidal problems have since been continuously carried out. It was early shown* that certain methods of analysis were incomplete and that the harmonic constituents taken into account do not fully represent the tidal oscillation. Special attention was called to the great increase required in the number of higher harmonics ("overtides" and "compound tides") needed to represent tidal oscillations in shallow water. An investigation† on the variations of "constants" obtained from yearly batches of observations provided additional evidence concerning the defects of analysis, and it was shown that the constants for important constituents are considerably perturbed by contributions from other constituents. In the year 1921‡ was published a very thorough expansion of the tide-generating potential, and many new constituents were indicated as being worthy of attention. Meteorological perturbations of sea-level and tides have also been studied,§ but much yet remains to be done, and such investigations require exact and complete methods of analysis.

A tidal record may be assumed to consist of three parts :—

(1) Oscillations of known periods and whose relative importance is known ;

(2) Oscillations whose periods are not known *a priori*, though exact periods may be deduced from considerations of causes, if such become known ;

(3) Oscillations of no persistent periodicity or amplitude and which may be regarded as sources of "casual errors," such as may be attributable to meteorological variations.

* 'Reports of Committee on Tides, British Association for the Advancement of Science,' 1920 and 1921.

† A. T. DOODSON, "Perturbations of Harmonic Tidal Constants," 'Roy. Soc. Proc.,' A, vol. 106, p. 513 (1924).

‡ A. T. DOODSON, "The Harmonic Development of the Tide-Generating Potential," 'Roy. Soc. Proc.,' A, vol. 100, p. 305 (1921).

§ A. T. DOODSON, "Meteorological Perturbations of Sea-Level and Tides," 'Monthly Notices R.A.S., Geophysical Supplement' (April, 1924).

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For analysis it is necessary that the observations be combined so as to magnify one constituent relatively to all others; in practice linear combinations only are used, preferably of hourly heights extending over a whole year, and magnification may be performed in two stages, the first of which uses a simple linear combination of hourly heights, and the second, essentially a correction process, uses the results of all the linear combinations for all constituents. An almost unlimited number of such methods may be devised, and in the absence of unknown and casual oscillations the results should be identical. The second type of oscillation has nearly always been ignored, but a great deal of attention has been given to methods professing to make the casual errors as small as possible. The author believes that the method of least squares has been applied too rigorously, to the detriment of the subject, and special consideration is given to this matter in § 3.

The methods of analysis in common use are as follows:—

(1) The B.A. methods, essentially as evolved by THOMSON, ROBERTS and DARWIN, and published in the 'Reports of the British Association for the Advancement of Science' between the years 1866 and 1885. These have been used by ROBERTS and the Survey of India.

(2) DARWIN's "Method for the Solar Constituents,"* and his methods based on the use of the "tidal abacus,"* both published in 1892. These have been extensively used throughout the world.

(3) The U.S.A. method,† first published in 1893, and chiefly used by the Coast and Geodetic Survey.

(4) BÖRGEN's Method,‡ 1894, principally used in Germany.

These methods may be judged by the following criteria:—

(*a*) The amount of labour involved;

(*b*) The degree of elimination of all other constituents in the analysis for a particular constituent;

(*c*) The completeness of the analysis.

The labour involved is greatest in the B.A. method, as it necessitates the re-writing of the hourly heights on special forms for each constituent; the use of the tidal abacus requires the hourly heights to be re-written once only, and the method for solar constituents further diminishes the total labour. The U.S.A. method, as first devised, uses some 2000 stencils in order to avoid copying hourly heights, and 24 sums for special hours are entered for each constituent for each week, so that it requires only

* G. H. DARWIN, "On an Apparatus for Facilitating the Reduction of Tidal Observations," 'Roy. Soc. Proc.,' vol. 52, p. 345 (1892).

† 'Report of the Tidal Division of the U.S. Coast and Geodetic Survey Office for the Fiscal Year ending June 30, 1893, Part I,' p. 108.

‡ C. BÖRGEN, "Über eine neue Methode, die harmonischen Konstanten der Gezeiten abzuleiten," 'Ann. der Hydrog.,' p. 219 (1894).

one-seventh the amount of writing required by the B.A. method; further labour is now saved by using the same weekly sets of hourly sums for two or more constituents whose speeds are very nearly multiples or sub-multiples of one another. BÖRGEN's method uses a continuous summation of the hourly heights at a fixed hour of the day, there being 24 such summations; for each constituent about 20 of these sums are picked out and form the material of analysis. The labour involved in this method is probably less than in any of the other methods.

The elimination of unwanted contributions is very imperfect in the B.A. method and in DARWIN's subsidiary methods, but corrections appear to be regularly made in the U.S.A. and BÖRGEN's methods. In both cases, however, the correction process is unnecessarily laborious (see § 10, where this point is discussed in connexion with BÖRGEN's method).

The completeness of the analysis cannot be adequately examined by any of the methods, and all presuppose the existence of certain harmonic constituents only. DARWIN's method for the solar constituents and the weekly sums of the U.S.A. method provide material for subsidiary examination or analysis, but only in a limited way.

The author has long considered DARWIN's method for the solar series as one of the greatest improvements hitherto made, and believes that the ideal method of analysis is one which carries out similar ideas for all constituents. The method now to be described may perhaps be regarded as the logical outcome of DARWIN's method, but it has wider claims to attention as it attempts to satisfy all the criteria mentioned above. It has certain relationships as regards principles with BÖRGEN's method (discussed in § 10), though it differs entirely in detail; and the author believes it to be less laborious, especially in connexion with the correction process, due to the choice of a central time-origin; also it possesses the advantages of yielding data suitable for tests on the completeness of analysis and for research purposes.

The method has been well tested at the Tidal Institute; over 30 years of tidal records from many parts of the world have been analysed by this method.

The general principles of the new method are very simple. Tidal constituents are divisible into "species" having 0, 1, 2 ... complete periods in approximately a solar day, and we may define S_0, S_1, S_2 ... as the solar constituents whose phase-increments are exactly 0°, 15°, 30° ... per mean solar hour, the suffix indicating the species. There are other constituents, K_1 and K_2, for instance, whose speeds differ so little from those of S_1 and S_2 that the corresponding constituents only separate in phase by 30° and 60° per month respectively. Now DARWIN saw that the analytical results for S_1 and S_2 from tidal observations extending over a month necessarily contained very large contributions from K_1 and K_2 respectively, and that these contributions would reveal themselves as annual and semi-annual perturbations of the monthly values for the constants obtained as for S_1 and S_2. By subsidiary analyses of sets of twelve monthly numbers he was able to determine functions pertaining only to S_1, others pertaining to K_1 only, and so on. The method was developed to obtain quite a large number of constituents simply related to S_0, S_1, S_2 .... *Whereas* DARWIN *only dealt with blocks of a month's observations, the author has considered units of a day.* Consequently, the hourly heights are dispensed with after analysing each day's observations as for S_0, S_1, S_2 .... For the semi-diurnal species of constituents two numbers, X_2, Y_2, are obtained, and the principal lunar constituent, M_2, perturbs X_2, Y_2 by amounts with a period of a fortnight. Further, the remainder of the semi-diurnal constituents fall into *groups* according to the periods of their perturbations of X_2, Y_2.

Thus the next stage of the analysis is to determine functions such as X_{pq}, Y_{pq}, where the first suffix refers to the species and the second one refers to the group; the second suffix indicates that the principal contributory terms are those which perturb X_p by a quantity with q complete periods per month.

Finally, if the months are properly chosen, each constituent in the group q contributes to the values of X_{pq} for consecutive months amounts varying with an integral number of periods, denoted by r, in approximately a solar year. By suitable combination of the values of X_{pq} a quantity X_{pqr} is obtained, and the principal contribution to it, in general, is that of a single constituent.

There is obviously condensation of the material; for example, commencing with 9000 hourly observations 360 values of X_2 are obtained, and for each month 9 values of X_{2q}, that is, 108 values of X_{2q} in all, and from these 37 values of X_{2qr} yield, after correction, harmonic constants for at least 18 known semi-diurnal constituents, and provide data which, by inspection only, indicate whether other constituents exist. If such constituents are indicated, the labour required to evaluate them is negligible. There is thus no temptation to restrict the analysis, as in the older methods, where the labour is almost directly proportional to the number of constituents.

The method is probably speedier in operation than any yet published, though comparisons as to time taken by any method are not readily available. One computer, commencing with tabulated hourly heights, performs the whole analysis for about 40 constituents in 10 working days of six hours each, and about one day in all is spent on sundry checks by other computers. The analysis for 20 more constituents would probably not take more than another six hours.

The correlation with astronomical arguments has been very much simplified; this section of the work of analysis has hitherto been very much overlaid with symbols, most of which are unnecessary. The author's paper* on the potential simplified the derivation of the harmonic terms, and the benefit is now conferred on the work of analysis.

The exposition is divided into two parts; in §§ 2 to 10 are found the theoretical considerations, followed by tables fundamental to the method but not required by computers; in §§ 11 to 20 are detailed instructions to computers without reference to theoretical considerations, and these instructions are followed by tables required in actual computations.

* *Loc. cit.*

2. *Notation.*

The following notation will be used :—

ζ = the tidal elevation above a known datum; it is usually tabulated at intervals of a mean solar hour.

σ = the increment of phase of a constituent, in degrees per mean solar hour.

ρ = the increment of phase in degrees per mean solar day with integral multiples of 360° omitted.

H = the hour of the day, in practice measured from midnight.

t = the number of hours measured from the origin of time.

T = the number of complete days measured from the origin of time.

$\overline{\mathrm{T}}$ = the number of the middle day of a specially defined month.

R = the amplitude of a constituent.

$\mathrm{R}\cos(\sigma t-\varepsilon)$ = a typical tidal constituent with phase of $-\varepsilon$ at the origin of time.

$\sum_c$ = a symbol for the aggregation of all constituents.

Σ = the usual symbol for arithmetical summation of a number of quantities.

X, Y = combinations of hourly heights.

A = X + Y.

B = X − Y.

d = multipliers for the daily values of X_p, Y_p.

m = multipliers for the monthly values of X_{pq}, Y_{pq}.

3. *The Fundamental Basis of Analysis.*

Practically all methods of analysis for harmonic constituents utilise in greater or less degree the well-known principles of the method of least squares.

Let the elevation ζ be composed of a number of constituents of type $\mathrm{R}\cos(\sigma t-\varepsilon)$, in which R and ε are unknown constants. An alternative form for each constituent is $\mathrm{A}\cos\sigma t+\mathrm{B}\sin\sigma t$. Then the Least Square Rule indicates that the "best values" of A and B are obtained from the limiting values, as N becomes infinite, of

$$\frac{2}{\mathrm{N}}\sum_{t=0}^{\mathrm{N}}\zeta\cos\sigma t,\qquad \frac{2}{\mathrm{N}}\sum_{t=0}^{\mathrm{N}}\zeta\sin\sigma t \quad \ldots\ldots\ldots\ldots \quad (3)$$

respectively.

In practice these limits are never reached, but fairly good approximations to A and B are obtained from observations covering a whole year, especially if N is carefully chosen. The computations for A and B have been usually performed in two stages :—

(I) *The Assignment*, in which the observations with approximately common values of $\cos\sigma t$ and $\sin\sigma t$ are grouped; thus for diurnal constituents with σ approximately equal to 15° those observations for which $\sigma t = n360° \pm 7°\cdot 5 + m15°$, where m and n are integers, are assigned to the value $\sigma t = m15°$ with $m \leq 24$.

(II) *The Harmonic Analysis*, in which the mean values of ζ in each group obtained from the assignment are multiplied by appropriate values of cos $m15°$, the results summed and divided by 12, so giving the desired value of A. Similar operations with sin $m15°$ yield B.

Casual errors are supposed to be minimised, and systematic errors due to imperfect elimination of other constituents require correction.

3.1. The harmonic analysis referred to is based on the rigid application of the Least Square Rule to the hourly means, and many elaborate forms for the computations have been devised, but DARWIN* advocated the use of General STRACHEY'S Rules, in which the Least Square Rule is not rigidly applied, but in which the old methods of grouping so as to diminish the multiplications required are still retained.

The author believes that much unnecessary labour and complicated calculations have resulted from strict adherence to the Least Square Rule, and that a great mistake has been made in considering casual errors to be more important than systematic errors. At the same time, he accepts the Rule as a safe theoretical guide, but has definitely abandoned the idea that it must be rigidly applied, in the sense that the coefficients used are to be exact to a high order. In tidal work it is only necessary to consider the 1st, 2nd, 3rd, 4th, 6th and 8th harmonics, and the mere subtraction of the 13th term from the 1st, the 14th from the 2nd, and so on, serves to eliminate the harmonics of even order, while addition serves to eliminate those of odd order. It has been found that if 2 cos σt, 2 sin σt are replaced by ± 2, ± 1, or 0, according to magnitude and sign, then very simple and convenient formulæ are obtainable, and that they can be used direct with the 24 mean values of ζ without having to make the elaborate combinations used hitherto to save multiplications. It is easy to use the formulæ mentally, and they are readily adapted to the use of a calculating machine. The method of analysis here described depends essentially on this simplification of the coefficients of the Least Square Rule.

3.2. We now proceed to write the Least Square Rule in a new form, which throws a great deal of light on the new method of analysis.

We have

$$\sigma t = \sigma \mathrm{H} + \rho \mathrm{T} = \sigma \mathrm{H} + \rho\,(\mathrm{T} - \overline{\mathrm{T}}) + \rho \overline{\mathrm{T}}, \qquad (3.21)$$

whence

$$\left.\begin{aligned}\zeta \cos \sigma t &= \zeta \cos \sigma \mathrm{H}\,\{\cos \rho\,(\mathrm{T}-\overline{\mathrm{T}}) \cos \rho\overline{\mathrm{T}} - \sin \rho\,(\mathrm{T}-\overline{\mathrm{T}}) \sin \rho\overline{\mathrm{T}}\} \\ &\quad - \zeta \sin \sigma \mathrm{H}\,\{\sin \rho\,(\mathrm{T}-\overline{\mathrm{T}}) \cos \rho\overline{\mathrm{T}} + \cos \rho\,(\mathrm{T}-\overline{\mathrm{T}}) \sin \rho\overline{\mathrm{T}}\} \\ \zeta \sin \sigma t &= \zeta \sin \sigma \mathrm{H}\,\{\cos \rho\,(\mathrm{T}-\overline{\mathrm{T}}) \cos \rho\overline{\mathrm{T}} - \sin \rho\,(\mathrm{T}-\overline{\mathrm{T}}) \sin \rho\overline{\mathrm{T}}\} \\ &\quad + \zeta \cos \sigma \mathrm{H}\,\{\sin \rho\,(\mathrm{T}-\overline{\mathrm{T}}) \cos \rho\overline{\mathrm{T}} + \cos \rho\,(\mathrm{T}-\overline{\mathrm{T}}) \sin \rho\overline{\mathrm{T}}\}\end{aligned}\right\} \qquad (3.22)$$

* 'Collected Papers,' vol. 1, p. 125.

Let ζ_c denote the sum of the products $\zeta \cos \sigma H$ for the 24 values of H in a day; then ζ_c is a function of T. Let ζ_{cc} denote the sum of the products $\zeta_c \cos \rho (T - \overline{T})$ for all values of T within a "month," with $\overline{T}$ as the number of the central day of this "month"; the result is a function of $\overline{T}$. Let ζ_{ccc} denote the sum of the products $\zeta_{cc} \cos \rho\overline{T}$ for the 12 equal and opposite values of $\overline{T}$.

Similarly, let processes involving sines be denoted by a suffix *s*.

Then, referring to § 3, we have :—

$$2A = 2R \cos \varepsilon = \text{average value of } (\zeta_{ccc} - \zeta_{css} - \zeta_{ssc} - \zeta_{scs}), \quad . \quad . \quad . \quad . \quad (3.23)$$

$$2B = 2R \sin \varepsilon = \text{average value of } (\zeta_{scc} - \zeta_{sss} + \zeta_{csc} + \zeta_{ccs}). \quad . \quad . \quad . \quad . \quad (3.24)$$

Except that multiplications are not by cosines and sines but by integers closely proportional to them, this is the essence of the Tidal Institute method.

It will be noted that if we take the mean values* of each of H, $T - \overline{T}$, $\overline{T}$ to be zero, then the mean values of $\cos \sigma H \sin \sigma H$, $\cos \rho (T - \overline{T}) \sin \rho (T - \overline{T})$, $\cos \rho\overline{T} \sin \rho\overline{T}$ are each zero. It is then readily shown that each of the terms on the right of (3.23) is proportional to $R \cos \varepsilon$, so that on division by the appropriate factors we have four different ways of obtaining this quantity. But further consideration shows that constituents with value of ρ defined by

$$\rho = \pm \rho_0 \pm r, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.25)$$

where $\rho_0\overline{T}$ is a multiple of 360° and r is small and is approximately an integral number of degrees, will be magnified almost equally though with different signs in the four functions, so that, in short, we have four functions to determine four different constituents. It is very unusual to have such a relationship between the values of ρ for four constituents, but frequently we have to consider two constituents *conjugate in this sense* one to the other.

The T.I. method of analysis uses functions analogous to those called ζ_{ccc}, ... above, and we see :—

(1) that the method is really an application of the Least Square Rule, with approximations to the coefficients indicated by the rule;

(2) that component functions may occur in sets of four and are usable for two or more constituents, conjugate in certain special ways;

(3) that if a constituent is expected to be the sole contributor to each of the four functions we have a valuable check upon either

(*a*) the computation; or

(*b*) the validity of the expectation;

in other words, we have a possible indication of unexpected constituents.

* The theory is independent of the precise origin of time; in practice H is measured from midnight and the theory is adapted to an implied change of time origin.

4. *Tidal Functions.*

Table I contains a list of tidal constituents with their values of σ and ρ, and examination shows that very approximately

$$\sigma = 15p\text{H}^\circ.$$
$$\rho = \pm q 12^\circ\cdot 19 \pm r^\circ.$$

where, p, q, r are integers. If $\overline{\text{T}}$ is chosen at intervals of $360/12\cdot19$ days, then $\rho\overline{\text{T}}$ is $\pm r\overline{\text{T}}$. In the sequel the values of $\pm\overline{\text{T}}$ are chosen as nearly as possible to satisfy this condition, and $\rho\overline{\text{T}}$ increases by nearly 30° per month if r is unity.

We now proceed to define certain symbols, with the understanding that in the numerical applications multiples of cosines and sines are replaced by integers ± 2, ± 1, 0.

Let

$$\text{A}_p = \Sigma\,\zeta\cos 15\,p\text{H},\qquad \text{B}_p = \Sigma\,\zeta\sin 15\,p\text{H},$$

the summations extending over 24 hourly heights; these correspond to ζ_c, ζ_s of § 3.2.

Using numerical suffixes to represent cosines, and literal suffixes to represent sines, let

$$\left.\begin{aligned} d_0,\ d_1,\ d_2, \ldots &= \text{values of } \cos\{-q\,12^\circ\cdot19\,(\text{T}-\overline{\text{T}})\} \\ d_a,\ d_b, \ldots &= \text{values of } \sin\{-q\,12^\circ\cdot19\,(\text{T}-\overline{\text{T}})\} \end{aligned}\right\}\quad (q = 0, 1, 2 \ldots),$$

$$\left.\begin{aligned} m_0, m_1, m_2, \ldots &= \text{values of } \cos r\overline{\text{T}} \\ m_a, m_b, \ldots &= \text{values of } \sin r\overline{\text{T}} \end{aligned}\right\}\quad (r = 0, 1, 2\ \ldots).$$

With $q = 1$, $r = 1$, we have

(i) with summations from $\text{T} - \overline{\text{T}} = -14$ to 14

$$\text{A}_{p1} = \Sigma\,d_1\text{A}_p,\qquad \text{A}_{pa} = \Sigma\,d_a\text{A}_p,\qquad \text{B}_{p1} = \Sigma\,d_1\text{B}_p,\qquad \text{B}_{pa} = \Sigma\,d_a\text{B}_p,$$

corresponding respectively to

$$\zeta_{cc},\qquad \zeta_{cs},\qquad \zeta_{sc},\qquad \zeta_{ss};$$

(ii) with summations for the 12 values of $\overline{\text{T}}$

$$\text{A}_{p11} = \Sigma\,m_1\text{A}_{p1},\quad \text{A}_{p1a} = \Sigma\,m_a\text{A}_{p1},\quad \text{A}_{pa1} = \Sigma\,m_1\text{A}_{pa},\quad \text{A}_{paa} = \Sigma\,m_a\text{A}_{pa},$$

corresponding respectively to

$$\zeta_{ccc},\qquad \zeta_{ccs},\qquad \zeta_{csc},\qquad \zeta_{css}.$$

Similarly we obtain functions

$$\text{B}_{p11},\quad \text{B}_{p1a},\quad \text{B}_{pa1},\quad \text{B}_{paa},$$

corresponding respectively to

$$\zeta_{scc},\quad \zeta_{scs},\quad \zeta_{ssc},\quad \zeta_{sss}.$$

The notation for functions arising from other values of q and r is similar.

Taking tidal constituents in the form

$$\Sigma\, \mathrm{R} \cos (\sigma t - \delta),$$

the results of the hourly multipliers may be written

$$\mathrm{A}_p = \sum_c a_p \mathrm{R} \cos (\delta - \rho \mathrm{T}) = \sum_c a_p \mathrm{R} \cos \{\delta - \rho (\mathrm{T} - \overline{\mathrm{T}}) - \rho \overline{\mathrm{T}}\},$$

$$\mathrm{B}_p = \sum_c b_p \mathrm{R} \sin (\delta - \rho \mathrm{T}) = \sum_c b_p \mathrm{R} \sin \{\delta - \rho (\mathrm{T} - \overline{\mathrm{T}}) - \rho \overline{\mathrm{T}}\},$$

whence we may write

$$\mathrm{A}_{p1} = \sum_c a_{p1} \mathrm{R} \cos (\delta - \rho \overline{\mathrm{T}}), \qquad \mathrm{B}_{p1} = \sum_c b_{p1} \mathrm{R} \sin (\delta - \rho \overline{\mathrm{T}}),$$

$$\mathrm{A}_{pa} = \sum_c a_{pa} \mathrm{R} \sin (\delta - \rho \overline{\mathrm{T}}), \qquad \mathrm{B}_{pa} = \sum_c b_{pa} \mathrm{R} \cos (\delta - \rho \overline{\mathrm{T}}).$$

If, for a single constituent, we write

$$\mathrm{D}_1 = \Sigma\, d_1 \cos \rho (\mathrm{T} - \overline{\mathrm{T}}),$$

$$\mathrm{D}_a = \Sigma\, d_a \sin \rho (\mathrm{T} - \overline{\mathrm{T}}),$$

where the summation extends from $\mathrm{T} - \overline{\mathrm{T}} = -14$ to 14, then

$$a_{11} = a_1 \mathrm{D}_1, \qquad b_{11} = b_1 \mathrm{D}_1,$$

$$a_{1a} = a_1 \mathrm{D}_a, \qquad b_{1a} = -b_1 \mathrm{D}_a.$$

Similarly, the results of the annual processes may be written as

$$\mathrm{A}_{p11} = \sum_c a_{p11} \mathrm{R} \cos \delta, \qquad \mathrm{B}_{p11} = \sum_c b_{p11} \mathrm{R} \sin \delta,$$

$$\mathrm{A}_{p1a} = \sum_c a_{p1a} \mathrm{R} \sin \delta, \qquad \mathrm{B}_{p1a} = \sum_c b_{p1a} \mathrm{R} \cos \delta,$$

$$\mathrm{A}_{pa1} = \sum_c a_{pa1} \mathrm{R} \sin \delta, \qquad \mathrm{B}_{pa1} = \sum_c b_{pa1} \mathrm{R} \cos \delta,$$

$$\mathrm{A}_{paa} = \sum_c a_{paa} \mathrm{R} \cos \delta, \qquad \mathrm{B}_{paa} = \sum_c b_{paa} \mathrm{R} \sin \delta,$$

with a similar notation for functions arising from other values of q and r.

If

$$\mathrm{M}_1 = \Sigma m_1 \cos \rho \overline{\mathrm{T}},$$

$$\mathrm{M}_a = \Sigma\, m_a \sin \rho \overline{\mathrm{T}},$$

where the summation is taken for the 12 values of $\overline{\mathrm{T}}$, then

$$a_{p11} = a_p \mathrm{D}_1 \mathrm{M}_1, \qquad b_{p11} = b_p \mathrm{D}_1 \mathrm{M}_1,$$

$$a_{p1a} = a_p \mathrm{D}_1 \mathrm{M}_a, \qquad b_{p1a} = -b_p \mathrm{D}_1 \mathrm{M}_a,$$

$$a_{pa1} = a_p \mathrm{D}_a \mathrm{M}_1, \qquad b_{pa1} = -b_p \mathrm{D}_a \mathrm{M}_1,$$

$$a_{paa} = -a_p \mathrm{D}_a \mathrm{M}_a, \qquad b_{paa} = -b_p \mathrm{D}_a \mathrm{M}_a.$$

Values of the daily multipliers d are given in Table XV, and values of the monthly multipliers m are given in Table XVI. A little freedom has been exercised in case of those coefficients which are approximately equal to $\pm 1\cdot 5$ or to $\pm\ 0\cdot 5$ in order to make small the contributions of some large constituent or of one whose value of ρ marks it out for special consideration.

It has been found, however, that the hourly multipliers corresponding roughly to $2 \cos 15^\circ pH, 2 \sin 15^\circ pH$ require modification for reasons discussed in the next paragraph, but the general principles remain unaffected.

5. *Derivation of Formulæ for Daily Processes.*

The first stage of the work of analysis is to construct linear combinations of hourly heights for each solar day. There are considerable advantages in having each of these, for all practical purposes, a function only of constituents of a single species, but to achieve this it is absolutely necessary to use formulæ extending beyond a solar day.

Let the contribution of a particular constituent to the height of tide at hour H be expressed by

$$\zeta_H = R \cos (\sigma H - \varepsilon + \rho T).$$

Then a linear combination such as

$$\zeta_H + \zeta_{H+2} + \zeta_{H+4}$$

is expressible in the form

$$JR \cos (\sigma H - \varepsilon + \rho T + \eta),$$

and the following table gives values of J and η for various linear combinations. In the case quoted J is $\sin 6\sigma/\sin 2\sigma$, and if σ is 30°, 60°, or 120° then $J = 0$; therefore, such a combination would contain zero contributions from constituents S_2, S_4, S_6 and small contributions from all semi-diurnal constituents. It is desired to assemble as many of these combinations as are necessary to isolate a single species.

TABLE of Linear Combinations.

No.	Combinations with $H = 0$.	J	η	Constituents with $J = 0$.
1	$\zeta_0 + \zeta_2$	$2 \cos \sigma$	σ	S_6
2	$\zeta_0 + \zeta_3$	$2 \cos 1\cdot 5\sigma$	$1\cdot 5\sigma$	S_4
3	$\zeta_0 + \zeta_4$	$2 \cos 2\sigma$	2σ	S_3
4	$\zeta_0 + \zeta_6$	$2 \cos 3\sigma$	3σ	S_2, S_6
5	$\zeta_0 + \zeta_{12}$	$2 \cos 6\sigma$	6σ	S_1
6	$\zeta_0 - \zeta_2$	$2 \cos (\sigma + 90^\circ)$	$\sigma + 90^\circ$	S_0
7	$\zeta_0 - \zeta_3$	$2 \cos (1\cdot 5\sigma + 90^\circ)$	$1\cdot 5\sigma + 90^\circ$	S_0, S_8
8	$\zeta_0 - \zeta_4$	$2 \cos (2\sigma + 90^\circ)$	$2\sigma + 90^\circ$	S_0, S_6
9	$\zeta_0 - \zeta_6$	$2 \cos (3\sigma + 90^\circ)$	$3\sigma + 90^\circ$	S_0, S_4, S_8
10	$\zeta_0 - \zeta_8$	$2 \cos (4\sigma + 90^\circ)$	$4\sigma + 90^\circ$	S_0, S_3, S_6
11	$\zeta_0 - \zeta_{12}$	$2 \cos (6\sigma + 90^\circ)$	$6\sigma + 90^\circ$	S_0, S_2, S_4, S_6, S_8
12	$\zeta_0 + \zeta_2 + \zeta_4$	$\sin 3\sigma/\sin \sigma$	2σ	S_4, S_8
13	$\zeta_0 + \zeta_4 + \zeta_8$	$\sin 6\sigma/\sin 2\sigma$	4σ	S_2, S_4, S_8
14	$\zeta_0 + \zeta_8 + \zeta_{16}$	$\sin 12\sigma/\sin 4\sigma$	8σ	S_1, S_2, S_4, S_8
15	$\zeta_0 + \zeta_9 + \zeta_{18}$	$\sin 13\cdot 5\sigma/\sin 4\cdot 5\sigma$	9σ	O_1 (nearly)
16	$\zeta_0 + \zeta_5 + \zeta_{10} + \zeta_{15} + \zeta_{20}$	$\sin 12\cdot 5\sigma/\sin 2\cdot 5\sigma$	10σ	Lunar series (nearly)

Using this table we find that

$$(\zeta_H - \zeta_{H+6}) + (\zeta_{H+2} - \zeta_{H+8}) + (\zeta_{H+4} - \zeta_{H+10})$$
$$= 2\cos(3\sigma + 90^\circ)(\sin 3\sigma/\sin\sigma)\,R\cos\{(\sigma H - \varepsilon + \rho T) + (3\sigma + 90^\circ + 2\sigma)\}.$$

We have obviously taken combinations (9) and (12), with ζ_0 in the former replaced by $\zeta_H + \zeta_{H+2} + \zeta_{H+4}$ and a corresponding change made in ζ_6. Such compound linear combinations involve multiplication of factors J and addition of phase-increments η.

5.1. It has been found desirable to use functions X, Y instead of A, B with the relationships

$$A = X + Y.$$
$$B = X - Y.$$

The formula chosen for X_2 is obtained as follows, the numbers denoting the heights at the respective hours.

$$\left\{\begin{array}{l}[(0+2+4)-(4+6+8)]+[(2+4+6)-(6+8+10)] \\ \quad +[(12+14+16)-(16+18+20)]+[(14+16+18)-(18+20+22)]\end{array}\right\}$$
$$-\left\{\begin{array}{l}[(6+8+10)-(10+12+14)]+[(8+10+12)-(12+14+16)] \\ \quad +[(18+20+22)-(22+24+26)]+[(20+22+24)-(24+26+28)]\end{array}\right\}.$$

Ordinary brackets contain combinations of type (12); two such combinations in square brackets contain the combinations (12) and (8); each line with two sets of square brackets contains combinations (12), (8), (1); each pair of lines in curl brackets adds the combination (5) and the two sets of curl brackets bring in the combination (9).

The total value of J is $32\cos\sigma\cos 6\sigma\sin^2 3\sigma$ and the total value of η is $14\sigma + 180^\circ$. A minus sign may be taken with J and 180° deleted from η. For S_2 the value of J is 24.

The following table exhibits the contributory factors and the values of J for certain constituents; only approximate values of σ are used.

——	σ.	$\lvert\cos\sigma\rvert$.	$\lvert\cos 6\sigma\rvert$.	$\lvert\sin 3\sigma\rvert$.	J/24.
M_2	29°	0·875	0·995	1·000	1·016
M_4	58	0·530	0·978	0·105	0·004
M_6	87	0·052	0·951	0·988	0·003
M_8	116	0·438	0·914	0·208	0·010
O_1	14	0·970	0·105	0·669	0·059
MO_3	43	0·731	0·208	0·777	0·089
M*f*	1	1·000	1·000	0·052	0·004

Many other combinations have been tried but have failed to reduce sufficiently constituents of species other than semi-diurnal. If combination (1) is necessary to reduce M_6, (8) is required for M_6 and the Long Period constituents, (12) for M_4 and M_8 and (9) for M*f*, M_4, M_8. It is impracticable to reduce further the diurnal constituents, but none of these have values of ρ identical with those of semi-diurnal constituents except in the case of K_1 and R_2, and the value of J for K_1 is extremely small. It is only for the analysis of short lengths of record that corrections for O_1 have to be made; its effects appear as a small variation in M_2 constants, with a period of a year.

5.2. It is unnecessary to illustrate the genesis of the remaining formulæ. The combinations used are summarised in the following table and the actual formulæ are given in Table XIV:—

Function.	Combinations.
X_0	1, 14, 16
X_1	1, 3, 4, 11, 11
X_2	1, 5, 8, 9, 12
X_3	8, 11, 14, 15
X_4	4, 4, 5, 7
X_6	5, 6, 13, 13

The formulæ for the functions Y are exactly the same as those for the corresponding functions X, save that they start with different values of H. The differences in phase between the results of X, Y for a solar constituent S have been made approximately 90°, and the initial values of H have been chosen so that odd, as well as even, values of H are used in one or other of the functions X_p, Y_p.

The reason for using negative values of H in X_1 is to simplify the operations required in the analysis of short lengths of records, especially in connexion with the correction of M_2 on account of O_1.

The formula for X_3 is rather complicated, due to the difficulty of adequately diminishing contributions from diurnal constituents, but the extra labour is offset by the fact that in practice there is no need for Y_3 to be evaluated, as there is no constituent S_3 of any importance.

There is complete elimination of all unwanted solar constituents and the following table illustrates the degree of reduction of important constituents in each species. Signs are here ignored in the values of J.

——	σ	X_0.	X_1.	X_2.	X_3.	X_4.	X_6.
		J/30.	J/18·9.	J/24.	J/28·9.	J/16.	J/36.
M_2	29°	0·0006	0·0004	1·016	0·0020	0·0018	0·0008
M_4	58	0·003	0·017	0·004	0·012	0·967	0·005
M_6	87	0·002	0·001	0·004	0·024	0·015	0·914
M_8	116	0·004	0·074	0·010	0·089	0·092	0·024
O_1	14	0·002	1·065	0·059	0·003	0·021	0·013
MO_3	43	0·007	0·052	0·089	1·101	0·074	0·015
Mf	1	0·972	0·018	0·003	0·005	0·026	0·017

These coefficients are such that we can regard the results of the daily processes as being functions only of constituents of a single species.

5.3. It is obviously convenient to have the formulæ for X and Y identical save for the initial hour, but the computations for the amplitudes and initial phases require the use of the functions $A = X + Y$, $B = X - Y$, though X and Y are retained until the last stages of the analysis.

In general, for a single constituent,

$$X = JR \cos(\rho T - \varepsilon + \eta),$$
$$Y = JR \cos(\rho T - \varepsilon + \eta'),$$

where J depends upon the type of the linear contribution and η, η' depend upon the initial values of H. We then define

$$A = aR \cos(\delta - \rho T), \qquad B = bR \sin(\delta - \rho T)$$

as used in § 4, with

$$a = 2J \cos \tfrac{1}{2}(\eta - \eta'), \qquad b = -2J \sin \tfrac{1}{2}(\eta - \eta'),$$
$$\delta = \varepsilon - \Delta, \qquad \Delta = \tfrac{1}{2}(\eta + \eta').$$

In certain cases we do not use Y, and it is convenient to write

$$X = xR \cos(\rho T - \varepsilon + \Delta).$$

The following table gives the formal values of a, b, x and Δ, Tables VIII to XIII give the numerical values referred to in § 4, and Table XXIV gives the numerical values of Δ.

Function.	Δ.	a, b or x.
X_0	19σ	$2 \cos\sigma \dfrac{\sin 12{\cdot}5\sigma}{\sin 2{\cdot}5\sigma} \dfrac{\sin 12\sigma}{\sin 4\sigma}$
A_1	$15{\cdot}5\sigma$	$64 \cos\sigma \cos 2\sigma \cos 3\sigma \sin^2 6\sigma \cos 2{\cdot}5\sigma$
B_1	$15{\cdot}5\sigma$	$-64 \cos\sigma \cos 2\sigma \cos 3\sigma \sin^2 6\sigma \sin 2{\cdot}5\sigma$
A_2	$15{\cdot}5\sigma$	$-64 \cos^2\sigma \sin^2 3\sigma \cos 6\sigma \cos 1{\cdot}5\sigma$
B_2	$15{\cdot}5\sigma$	$64 \cos^2\sigma \sin^2 3\sigma \cos 6\sigma \sin 1{\cdot}5\sigma$.
X_3	25σ	$-2 \sin 6\sigma \dfrac{\sin 12\sigma}{\cos 2\sigma} \dfrac{\sin 13{\cdot}5\sigma}{\sin 4{\cdot}5\sigma}$
A_4	$15{\cdot}5\sigma - 90°$	$32 \sin 1{\cdot}5\sigma \cos^2 3\sigma \cos 6\sigma \cos\sigma$
B_4	$15{\cdot}5\sigma - 90°$	$-32 \sin 1{\cdot}5\sigma \cos^2 3\sigma \cos 6\sigma \sin\sigma$
X_6	$15\sigma + 90°$	$-4 \sin\sigma \cos 6\sigma \dfrac{\sin^2 6\sigma}{\sin^2 2\sigma}$

5.4. There is no simple connexion between the formulæ for X, Y and the Least Square Rule, but it may be noted that for the constituent S_2, where ζ repeats itself after

24 hours, the formula for X_2 reduces to a 12-term formula which is exactly that given by the Least Square Rule, with multiples of cosines and sines replaced by integers. Similar results hold for other formulæ.

For all practical purposes the formulæ give equal weight to all the hourly heights, though such is not vitally important. The function X_2, for instance, is based only on values of ζ for even values of H, and it might appear that the data were not fully used. It should be borne in mind that the casual errors (due largely to meteorological causes) are highly correlated from hour to hour but not from day to day, so that there is little or nothing to be gained by increasing the number of observations per day. DARWIN satisfied himself that for his processes the results from observations at intervals of two hours were practically the same as those from observations at intervals of an hour.

Coefficients of 2 and 4 with positive and negative signs may appear to be troublesome, but the method is intended for use with adding machines, and no difficulties have been encountered, even by inexperienced computers.

6. *Elimination Formulæ.*

After the completion of the annual process we have a number of functions of type A_{pqr}, B_{pqr}. While each of these contains a predominant contribution from a single constituent or from constituents conjugate in the sense of (3.25), it is necessary to eliminate the contributions by other constituents. It is the special virtue of the central time origin that each function contains either contributions in terms of R cos δ or of R sin δ, but not of both together. A linear combination of the functions can thus be found which will eliminate all unwanted contributions. An example for the semi-diurnal species will most readily explain the method.

The function A_{200} contains the following multiples of R cos δ :—

$$11812 \text{ for } S_2, \quad -223 \text{ for } M_2, \quad -332 \text{ for } K_2, \ \ldots,$$

and the functions A_{220}, A_{202} pertain principally to M_2 and K_2 respectively, the multiples of R cos δ being 13059 and 6395 respectively. Thus

$$(A_{200} + 0\cdot017A_{220} + 0\cdot052A_{202} + \ldots) \div 11812$$

gives the value of R cos for S_2. Table XXI gives this information in a compact form, the only difference noticeable being in the multiple of A_{202} due to the allowance made for contributions by K_2 to the other functions A_{201}, A_{211}, ... A considerable amount of labour has been required to deduce these linear combinations, successive approximation having been used.

Tables XVII to XXII contain the formulæ for the elimination process for the various sets of functions. It has been considered unnecessary to include correction terms arising from constituents which have small amplitudes.

An asterisk attached to the symbol for a constituent indicates that conjugate constituents have to be considered. Thus the functions for the "principal constituent T_2*" have to be used for R_2 also.

The amount of work involved in these corrections is not so much as the tables suggest, but satisfaction on this point can only be obtained by studying the example and the instructions to computers. In any case, no method hitherto used or likely to be invented can dispense with these eliminations, and the author claims that he has reduced this necessary labour to the minimum, and that the elimination formulæ are more exact and easier to apply than any yet published; those hitherto used have been first order approximations and have necessitated reference to trigonometrical tables for every term.

7. *Modification of Procedure.*

It is unfortunate that $\overline{T}$ proceeds at irregular intervals, for the values of X_{pq} obtained from 12 successive months cannot be checked by considerations of smoothness, and an alternative order of procedure is recommended in actual calculations.

The function X_{pqr} can be obtained from the function X_p by performing the annual process before the monthly process; that is, in practice the multipliers m are used for the entries in each row of the table of X_p, so yielding 29 values denoted by $X_{p.r}$, and then the multipliers d are used to give X_{pqr}. The advantage is that the 29 values of $X_{p.r}$ proceed by unit increments of $T - \overline{T}$ and smoothness tests are available, and that the labour of computation is considerably decreased because the number of third suffixes (r) is usually smaller than the number of second suffixes (q).

No alternative choice of $\overline{T}$ has all-round advantages to offer, the best alternative being to take it at intervals of 32 days; the calculations would be increased because of the greater number of values of suffix q, but the choice might be a suitable one for the analysis of observations extending over six months only.

8. *Examination of the Results.*

The method has been tested by Mr. W. A. D. SMITH† using accurate values of hourly heights computed some years ago in the Institute for research on tidal observations at Newlyn. As would be expected, there was strict agreement between the four values of $R \cos \delta$ given by the four functions referred to at the end of § 3.2. In general, however, we have no right to expect exact agreement, even if we do not suspect the existence of conjugate constituents, for the casual errors are rarely eliminated completely.

† The author is much indebted to Mr. SMITH, lately on the staff of the Department of Applied Mathematics in the University, for thorough tests of many of the tables, and especially for assistance in connexion with the modification of procedure, § 7. Many thanks are also rendered to the staff of the Tidal Institute for help received in the construction of the tables.

Taking the standard deviation of the meteorological perturbations of sea level to be 0·5 foot, then, if the perturbations from hour to hour were absolutely uncorrelated, we should expect the standard deviation of any average, whether taken with positive signs or with mixed positive and negative signs to be $0 \cdot 5/\sqrt{7{,}000}$ feet, or about 0·006 feet. But this we cannot expect, and we can only assume that the daily mean values of the meteorological perturbations are uncorrelated so that the standard deviation of any average would be about $0 \cdot 5/\sqrt{360}$ feet or 0·027 feet. Therefore, with meteorological perturbations of this order we should expect four values of R cos δ to differ from their mean by amounts of the order of 0·03 feet. For most places, the meteorological perturbations decrease as the species number p increases, so that long-period constituents are more affected than sixth diurnal constituents, as may be seen numerically from the run of X_p, Y_p.

In certain cases, however, it may appear that the discrepancies between the four values of H cos δ for a particular constituent are greater than those for other constituents of the same species. This was shown very markedly in the analysis of tidal observations for Vancouver; the conjugate constituents MP_1 and SO_1 had been ignored up to that time, but they revealed themselves as perturbations of O_1; the four values of R cos δ for O_1 were − 0·621, − 0·430, − 0·897, − 0·603 feet, and the four values of R sin δ were − 0·618, − 1·144, − 1·313, − 0·991 feet. All the diurnal constituents seemed to be perturbed, but the perturbations were only of the order of 0·03 feet. Analyses for a later year confirmed the existence of the conjugate constituents.

Predictions for Vancouver had been found to be unsatisfactory, and the presence of the constituents mentioned partly accounts for the errors of prediction; it is some degree of satisfaction to know the cause even though it may not be possible at an early date to incorporate mechanism for such new constituents on a tide-predicting machine.

The 29 values of $X_{p.r}$ referred to above may be used for research work, as all known contributions can be set up on a tide-predicting machine and their sum obtained from a run of 29 days. The residue may be examined graphically for real periodicities.

9. *Choice of Constituents.*

An attempt has been made to include in the method all constituents likely to be of importance. The choice depends upon

(1) The author's development of the tide-generating potential;
(2) His researches on shallow-water effects.

Free use has been made of a valuable memoir by Dr. H. Rauschelbach,* which itself uses the results of (1) and (2).

* Dr. H. Rauschelbach, "Harmonische Analyse der Gezeiten des Meeres—Eine Weiterentwicklung des Borgenschen Verfaturens," 1. Teil, 'Archiv der Deutschen Seewarte,' XLII, Hamburg (1924).

A special notation for constituents and arguments is used in the paper on the potential, to which reference should be made, if necessary.

In shallow water the second order terms of the dynamical equations have to be taken into account; these involve gradients of the squares and products of the components of velocity, and consideration of the harmonic development of these leads to a close approximation to the relative magnitude of the shallow-water constituents. Essentially, we get M_2 and S_2 yielding constituents M_4, MS_4, S_4, where the speeds of M_4 and S_4 are twice those of M_2 and S_2 respectively, while the speed of MS_4 is the sum of the speeds of M_2 and S_2; the amplitudes are respectively proportional to M_2^2, $2M_2S_2$, S_2^2. From these again by interaction we get sixth diurnal constituents M_6, $2MS_6$, $2SM_6$, S_6, and the notation $2MS_6$ means that the speed is twice that of M_2, plus the speed of S_2. By the same interaction semi-diurnal constituents such as $2MS_2$ are generated, with a speed $2M_2 - S_2$, using temporarily an obvious notation. Similarly, with other constituents we get

$$MNS_6, \text{ with speed } = M_2 + N_2 + S_2.$$

$$MNS_2, \quad ,, \quad = M_2 + N_2 - S_2.$$

$$MSN_2, \quad ,, \quad = M_2 + S_2 - N_2.$$

These examples serve to illustrate the notation.

In many instances a shallow-water constituent may have the same speed as a normal constituent of the potential; thus $2MS_2$ is identical with μ_2. Dr. RAUSCHELBACH has used a shallow-water notation for all constituents hitherto unnamed, but this cannot be accepted as ideal, as it sometimes gives a false notion as to the principal source of the constituent. His notation and that used in this method of analysis are set out below.

9.1. *The Long-Period Constituents* have not been specially considered. The constituents S*a*, S*sa*, M*m*, MS*f*, M*f* are usually taken into account, but they are all subject to such great meteorological perturbations that little reality can be attached to the results of even the best analyses, so far as the monthly and fortnightly constituents are concerned.

9.2. *The Diurnal Constituents* indicated in the paper on the potential as being worthy of consideration include seven not named up to that time. The constituents O_1, K_1, P_1, Q_1, M_1, J_1, OO_1, $2Q_1$, S_1 are commonly used, and two others, denoted by ρ_1 and σ_1, had been considered previously to the author's development of the potential, and he has seen no reason to change the notation.

In the following table

P is the notation used in the expansion of the potential, slightly modified to give arguments of cosines;

R gives RAUSCHELBACH's notation;

D is the notation proposed by the author.

The arguments are given partly in terms of the arguments of well-known constituents, and partly in terms of h, p, p_1, the mean longitudes of the sun, the lunar and solar perigrees respectively. The coefficients are taken from the paper on the potential, the corresponding coefficient for O_1 being 0·3769.

P.	Argument.	Coeff.	R.	D.
162·556 − 90°	(arg. P_1) $- h + p_1$	0·0103	TK_1	π_1
167·555 + 90	(arg. K_1) $+ 2h$	0·0076	KP_1	ϕ_1
173·655 + 90	(arg. J_1) $- 2h + 2p$	0·0057	λO_1	θ_1
157·455 + 90	(arg. M_1) $+ 2h - p$	0·0057	LP_1	χ_1
183·555 + 90	(arg. S_2) − (arg. O_1)	0·0049	SO_1	SO_1
147·555 + 90	(arg. O_1) $+ 2h + 180°$	0·0049	MP_1	MP_1
164·556 + 90	(arg. S_1) $+ p_1$	0·0042	S_1	S_1
166·554 + 90	(arg. K_1) $+ h - p_1$	0·0042	RP_1	ψ_1

Shallow-water constituents have their relative importance indicated by the following table, which is simply prepared from the products of the amplitudes of the constituents generating them; thus MK_1 is derived from M_2 and K_1 with a coefficient equal to the coefficient of M_2 multiplied by the coefficient of K_1.

—	K_1	O_1	P_1	Q_1	J_1
M_2	(0·48)	(0·34)	0·16	(0·06)	0·03
S_2	(0·22)	0·16	(0·08)	0·03	—
N_2	(0·09)	0·07	0·03	—	—
K_2	0·06	0·04	0·02	—	—

Those shallow-water constituents which perturb well-known normal constituents are indicated by brackets. The principal constituents remaining are MP_1 and SO_1 and possibly NO_1. Regarded as shallow-water constituents KP_1, λO_1, RP_1 are extremely small and these symbols for constituents indicated by the potential must be rejected.

In the lists of constituents in the tables those denoted by θ_1, ϕ_1, ψ_1 occur together, π_1 is close to P_1 and χ_1 is next to it.

The notation SO_1, MP_1 has been used for two constituents indicated by the potential, the reason being that if such constituents are of considerable importance, they will be due to shallow-water effects.

9.3. *Semi-Diurnal Constituents* denoted by M_2, S_2, N_2, K_2, ν_2, μ_2, L_2, T_2, $2N_2$, λ_2, R_2 have well-accepted symbols. In shallow water a large number of constituents are generated, some from semi-diurnal constituents only, as MSN_2, and others from diurnal constituents only. Leaving out of consideration the constituents which perturb the well-known constituents listed above, we have the following list; constituents on the same line have the same speed but different derivations :—

P.	Symbol.	Relative Coeff.	Symbol.	Relative Coeff.	D.
	MSN_2	0·69	—	—	MSN_2
227·655	MNS_2	0·69	—	—	MNS_2
	MSK_2	0·44	OP_2	0·074	OP_2
	MKS_2	0·44	—	—	MKS_2
285·455	MKN_2	0·19	KJ_2	0·010	KJ_2
	MNK_2	0·19	OQ_2	0·032	OQ_2
	$2SN_2$	0·15	—	—	—

If all these constituents were generated in shallow water and only from semi-diurnal constituents, it would probably be necessary only to consider two or three; the rest could be inferred by using the laws deduced from the dynamical equations. Hitherto very little attention has been paid to shallow-water constituents arising from the diurnal species, and the author's judgment is that constituents considered as OP_2, OQ_2, KJ_2 are likely to be of more interest and importance than if we consider them as MSK_2, MKS_2, MKN_2.

9.4. *Shallow-Water Constituents* in general offer a wide choice. Experience has definitely shown that unless we are prepared to deal with an extremely large number of constituents, we cannot expect to represent at all accurately the tidal oscillation in rivers such as the Thames or Mersey; especially in the case of the former we have to conclude that the harmonic method begins to be unusable for two reasons:—

(1) The tidal oscillation cannot be represented by harmonics limited to the sixth or eighth order, as the " convergence " is very slow;

(2) The number of terms of approximately equal importance within each species increases with the species number.

As a rough working rule the author has concluded that if eighth diurnal constituents cannot be neglected, it is hopeless to attempt to deal with them, while as regards quarter and sixth diurnal constituents it is concluded that only a few representatives, well scattered according to speed, need be chosen as the rest may be inferred.

Taking Dr. RAUSCHELBACH's numerical values for the relative importance of the constituents *within the species*, we get:—

Const.	Rel. Coeff.	Const.	Rel. Coeff.	Const.	Rel. Coeff.
MK_3	1·00	M_4	1·00	$2MS_6$	1·00
MO_3	0·69	MS_4	0·95	M_6	0·71
SK_3	0·48	MN_4	0·38	MSN_6	0·49
SO_3	0·33	MK_4	0·26	$2SM_6$	0·48
M_3	0·19	S_4	0·22	$2MN_6$	0·41
MQ_3	0·13	SN_4	0·18	$2MK_6$	0·28
NO_3	0·11	SK_4	0·12	MSK_6	0·26
SP_3	0·16	—	—	—	—
K_3	0·13	—	—	—	—

The notation MO_3 is preferable to that of $2MK_3$ as being more direct; the latter notation implies a derivation from M_4 and K_1, but we may have MO_3 occurring when the interaction of M_4 and K_1 is much too small to be considered. The first five of the third diurnal terms have been included in the analysis.

All the quarter diurnal terms given above have been included, but constituents such as $3MS_4$ arising with, or at the same time as, the eighth diurnal constituents have been ignored.

9.6. A list of constituents is given in Tables XXVII and XXVIII. The "astronomical argument" is the argument of the principal term of the corresponding constituent of the tide-generating potential and it is a linear function of

$$t,\ s,\ h,\ p,\ \mathrm{N},\ p_1,$$

where t is the time and s, h, p, N, p_1 are mean longitudes of the moon, sun, moon's perigee, moon's ascending node and solar perigee respectively. The last three variables increase very slowly, and a number of terms with common multiples of t, s, h must be combined to give a "constituent," written as

$$f\mathrm{H}\ \cos\,(\mathrm{V} + u - \kappa),$$

where V is the phase of the principal term and f, u are slowly varying functions, chiefly of N, but in some cases of p also; H and κ are the harmonic constants sought.

The analysis requires only the values of V at zero hour ($t = 0$) and the required expressions in terms of s, h, p and constant angles are given in Table XXVII. A separate table is given for compound constituents in terms of the generating constituents.

The values of f and u can be readily obtained from the harmonic development of the potential through $f \cos u$, $f \sin u$. Thus the complex constituent in L_2 has certain component terms as follows, where V is the argument of the principal term and $\mathrm{G}\cos^2\lambda$ is a "geodetic coefficient":

$$[0\cdot002567 \cos \mathrm{V} - 0\cdot000095 \cos\,(\mathrm{V} + \mathrm{N}) - 0\cdot000643 \cos\,(\mathrm{V} + 2p)$$
$$- 0\cdot000283 \cos\,(\mathrm{V} + 2p - \mathrm{N}) - 0\cdot000040 \cos\,(\mathrm{V} + 2p - 2\mathrm{N})$$
$$+ 0\cdot000012 \cos\,(\mathrm{V} + 2p + \mathrm{N})]\ \mathrm{G} \cos^2 \lambda,$$

and if these are expressed as

$$0\cdot002567\mathrm{G}\cos^2\lambda\,.\,f\cos\,(\mathrm{V} + u),$$

we get the expressions given for $f \cos u$, $f \sin u$ in Table XXVI.

Similar formulæ for $f \cos u$, $f \sin u$ in terms of N only have been obtained for other constituents, and have been used to verify DARWIN's formulæ (Table XXVI) for f and u as obtained from the analysis of tables of these quantities.

The constituent M_1 has two principal terms whose arguments differ from that of the true M_1 by $+p$ and $-p$. The combination, like that for L_2, is regrettable, as the increment in p is about 40° per annum, but it cannot be avoided. The correct formulæ for $f \cos u$, $f \sin u$ have been multiplied by an arbitrary factor in order to preserve continuity with DARWIN's values; certain errors were made by him which he afterwards discovered but allowed to stand.

The formulæ for L_2 and M_1 have been tested against tables published elsewhere and the agreement for L_2 is good; for M_1 a difference of 2° is possible, but such a difference is negligible with so small a constituent.

The formulæ for f and u are given in terms of p and N only, so that the author has succeeded in sweeping away the following variables:—

$$I, \xi, \nu, \nu', \nu'', R, Q, P.$$

A constant value of p_1 has been taken, as for the epoch 1950. Consequently, only the variables s, h, p, N remain to be used by the computer, and formulæ are given in Table XXV.

The terms of the constituent L_2 referred to above have a common geodetic coefficient $G \cos^2 \lambda$, where λ is the latitude of the place. We can assume that the corresponding tidal terms will all have the same phase-lag, but we cannot assume the same lag for certain terms of L_2 with a different geographical distribution of force. The author, in his paper on perturbations of harmonic constants, demonstrated this for N_2. Consequently, at the present time we have to omit these terms from f and u, though they are of equal importance with those taken into account.

For some of the smaller constituents slight changes have been made in f and u so as to use the values obtained for more important constituents.

Finally, the computer is urged to compute phase lags denoted by g instead of those denoted by κ; reference to § 20 should be made on this matter.

10. BÖRGEN'S *Method of Analysis.*

The present method of analysis had been in use for some time before the author realised that there are points of similarity with BÖRGEN's method so far as essential underlying principles are concerned. The comparison is best made in terms of the notation used in this paper.

We have given two ways of obtaining the function X_{pqr}; in both of these the daily process is used first to give X_p and the alternative procedures are to take next either the monthly process yielding X_{pq} or the annual process yielding $X_{p\cdot r}$. A third alternative would be to apply the monthly process first to the direct sequences of hourly heights, and to obtain monthly sets of 24 hourly sums corresponding to the suffix $\cdot q$, and from these to obtain sets of 24 hourly sums corresponding to suffixes $\cdot qr$. It will be clear that the numeral and literal suffixes would yield with $q = 1$, $r = 1$ the four functions $X_{.11}$, $X_{.1a}$, $X_{.a1}$, $X_{.aa}$. If, however, the daily multipliers were continued throughout the year, constituents such as N_2 and ν_2, which are conjugates in the sense of (3.25), would each have their sets of daily multipliers and each constituent would yield two sets of hourly sums.

10.1. The hourly sums would have to be analysed by formulæ such as those for X_p, Y_p *but limited to* 24 *hours*. In order to isolate contributions from a particular species we found it necessary for the formulæ to be extended outside the 24 hours. Consequently, by the procedure outlined above, the corrections for any given constituent would have to be made for all other constituents, and not for those of the same species only.

10.2. Such in essence is Börgen's method, but the daily multipliers are replaced by ± 1, and so the principles of the Least Square Rule are still further ignored. There is probably no real objection to this simplification, and, in fact, the author has supplied the Hydrographer with a method of analysis of tidal observations extending over 29 days, in which the multipliers for both daily and hourly processes are ± 1.

10.3. The instructions by Dr. Hessen,* however, imply that only one out of the two possible sets of 24 hourly sums is evaluated except when known conjugate constituents with equal and opposite values of ρ necessitate both. This simplification is equivalent to the arbitrary choice of half the functions X_{pqr} provided by the T.I. method. To compute all the possible functions by Börgen's method would approximately double the work, whereas in the T.I. method there is very little temptation to compute only half the functions, as the final processes are very simple and easily performed, owing to the condensation of the material at each stage.

10.4. A more serious modification still is that the daily multipliers ± 1 are not closely representative of the signs of cos ρT, sin ρT owing to the use of crude approximations to the values of ρ. Thus for M_2 the value of ρ is taken as $-24°$ instead of $-24°\cdot 38$; there is cumulative loss of angle and consequently a partial elimination of M_2 as well as of unwanted constituents. Since corrections are adequately applied, this means that the casual errors are relatively greater than they need to be under more ideal conditions. Much more serious simplifications appear to be made, if Dr. Hessen's exposition is correctly understood by the author, in the cases of O_1 and other constituents; the value of ρ for O_1 is taken as $360/8\cdot 4$ instead of $360/25\cdot 7$, that is, the period of its perturbation of X_1 is taken as 43 days instead of 14, and therefore only one-third of its full contribution is realised. Another simplification of doubtful validity is made with most of the shallow-water constituents; thus for M_6 only 6 days out of 64 appear to be utilised. It has been already remarked that casual errors are less evident with the shorter period constituents than with the long period constituents, and in the T.I. method only X_6 is used instead of X_6 and Y_6, but its value on every day is utilised.

* "Über die Börgensche Methode der Harmonischen Analyse . . .," 'Ann. der Hydrog.,' vol. 48, pp. 73, 123 and 177.

10.5. In contrast to all this the values of cos σh, sin σh, used in the analysis of 24 hourly sums, are used with the exact values of σ appropriate to the constituent. No doubt many of these simplifications and anomalies are due to the desire to have everything expressed in trigonometrical form, whereas in the T.I. method the data are all in the numerical form. One would judge, however, that labour might have been more profitably expended on the earlier processes of analysis.

10.6. It is necessary to remark that the simplicity of interpretation of the four functions referred to above and in § 3.2 is lost by the use of non-central origins of time, both in the daily process and in the computations corresponding to the monthly process, and for the same reason the corrections require much reference to trigonometrical tables.

10.7. While making these remarks and criticisms the author desires to place on record his opinion that Börgen's method is a very ingenious one, and admirable in many respects. If he had used the method as a model he would have used central time origins, more accurate values of ρ, monthly and annual processes (incidentally simplifying the difficulties regarding ρ), and numerical methods for the fundamental data; but the difficulties mentioned in § 10.1 would probably have been deterrent.

Table I.—List of constituents, with increments in phase in degrees per mean solar hour and per mean solar day.

—	σ.	ρ.	—	σ.	ρ.	—	σ.	ρ.
S_0	0·0000000	0·000000	OQ_2	27·3416964	—63·799285	MN_4	57·4238337	—61·827990
Sa	0·0410686	0·985647	MNS_2	27·4238337	—61·827990	M_4	57·9682084	—48·762998
Ssa	0·0821373	1·971295	$2N_2$	27·8953548	—50·511484	SN_4	58·4397295	—37·446491
Mm	0·5443747	13·064993	μ_2	27·9682084	—48·762998	MS_4	58·9841042	—24·381499
MSf	1·0158958	24·381499	N_2	28·4397295	—37·446491	MK_4	59·0662415	—22·410204
Mf	1·0980331	26·352793	ν_2	28·5125831	—35·698005	S_4	60·0000000	0·000000
—	—	—	OP_2	28·9019669	—26·352793	SK_4	60·0821373	1·971295
$2Q_1$	12·8542862	—51·497131	M_2	28·9841042	—24·381499	—	—	—
σ_1	12·9271398	—49·748645	MKS_2	29·0662415	—22·401204	$2MN_6$	86·4079380	—86·209489
Q_1	13·3986609	—38·432139	λ_2	29·4556253	—13·064993	M_6	86·9523127	—73·144496
ρ_1	13·4715145	—36·683652	L_2	29·5284789	—11·316506	MSN_6	87·4238337	—61·827990
O_1	13·9430356	—25·367146	T_2	29·9589333	— 0·985600	$2MS_6$	87·9682084	—48·762998
MP_1	14·0251729	—23·395851	S_2	30·0000000	0·000000	$2MK_6$	88·0503457	—46·791703
M_1	14·4920521	—12·190749	R_2	30·0410667	0·985600	$2SM_6$	88·9841042	—24·381499
χ_1	14·5695476	—10·330859	K_2	30·0821373	1·971295	MSK_6	89·0662415	—22·401204
π_1	14·9178647	— 1·971248	MSN_2	30·5443747	13·064993	—	—	—
P_1	14·9589314	— 0·985647	KJ_2	30·6265120	15·036287	—	—	—
S_1	15·0000000	0·000000	$2SM_2$	31·0158958	24·381499	—	—	—
K_1	15·0410686	0·985647	—	—	—	—	—	—
ψ_1	15·0821353	1·971248	MO_3	42·9271398	—49·748645	—	—	—
ϕ_1	15·1232059	2·956942	M_3	43·4761563	—36·572248	—	—	—
θ_1	15·5125897	12·302153	SO_3	43·9430356	—25·367146	—	—	—
J_1	15·5854433	14·050640	MK_3	44·0251729	—23·395851	—	—	—
SO_1	16·0569644	25·367146	SK_3	45·0410686	0·985647	—	—	—
OO_1	16·1391017	27·338441	—	—	—	—	—	—

TABLE II.—Values of cos $\rho\,(T - \overline{T})$ and sin $\rho\,(T - \overline{T})$ for diurnal constituents.

$T-\overline{T}$.	$2Q_1$.	Q_1.	ρ_1.	O_1.	M_1.	χ_1.	K_1.	ϕ_1.	θ_1.	J_1.	OO_1.
					Values of cos $\rho\,(T - \overline{T})$.						
0	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000
1	0·623	0·784	0·802	0·904	0·978	0·984	0·999	0·999	0·977	0·970	0·888
2	—0·225	0·228	0·286	0·633	0·911	0·936	0·999	0·995	0·909	0·882	0·578
3	—0·903	—0·427	—0·343	0·240	0·803	0·857	0·999	0·988	0·800	0·741	0·139
4	—0·899	—0·897	—0·836	—0·199	0·658	0·751	0·998	9·979	0·653	0·556	—0·331
5	—0·217	—0·977	—0·998	—0·600	0·486	0·620	0·997	0·967	0·477	0·338	—0·728
6	0·629	—0·635	—0·765	—0·885	0·290	0·470	0·995	0·952	0·279	0·099	—0·961
7	1·000	—0·017	—0·229	—0·999	0·081	0·304	0·993	0·935	0·068	—0·145	—0·980
8	0·616	0·608	0·398	—0·921	—0·131	0·128	0·991	0·916	—0·146	—0·381	—0·780
9	—0·233	0·970	0·867	—0·665	—0·337	—0·052	0·989	0·894	—0·354	—0·594	—0·406
10	—0·906	0·911	0·993	—0·281	—0·529	—0·230	0·986	0·870	—0·545	—0·772	0·059
11	—0·895	0·458	0·725	0·157	—0·696	—0·401	0·983	0·843	—0·711	—0·903	0·511
12	—0·208	—0·194	0·170	0·565	—0·831	—0·559	0·979	0·814	—0·845	—0·980	0·849
13	0·636	—0·761	—0·452	0·864	—0·930	—0·698	0·975	0·783	—0·939	—0·999	0·997
14	1·000	—0·999	—0·896	0·996	—0·987	—0·815	0·971	0·750	—0·991	—0·958	0·922
					Values of sin $\rho\,(T - \overline{T})$.						
0	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
1	—0·783	—0·621	—0·597	—0·428	—0·211	—0·179	0·017	0·051	0·213	0·243	0·459
2	—0·974	—0·974	—0·958	—0·775	—0·413	—0·353	0·034	0·103	0·416	0·471	0·816
3	—0·430	—0·905	—0·939	—0·970	—0·596	—0·515	0·051	0·154	0·601	0·671	0·990
4	0·438	—0·443	—0·549	—0·980	—0·752	—0·660	0·068	0·205	0·757	0·831	0·944
5	0·976	0·210	0·060	—0·800	—0·874	—0·784	0·085	0·255	0·879	0·941	0·686
6	0·778	0·772	0·644	—0·466	—0·957	—0·883	0·102	0·305	0·960	0·995	0·275
7	—0·008	1·000	0·974	—0·042	—0·997	—0·953	0·119	0·353	0·998	0·989	—0·197
8	—0·788	0·794	0·917	0·390	—0·991	—0·992	0·136	0·401	0·989	0·925	—0·625
9	—0·973	0·245	0·498	0·747	—0·941	—0·999	0·153	0·448	0·935	0·804	—0·914
10	—0·423	—0·412	—0·119	0·960	—0·849	—0·973	0·170	0·493	0·838	0·636	—0·998
11	0·446	—0·889	—0·689	0·988	—0·718	—0·916	0·187	0·537	0·703	0·430	—0·860
12	0·978	—0·981	—0·985	0·825	—0·555	—0·829	0·204	0·580	0·535	0·198	—0·529
13	0·772	—0·648	—0·892	0·503	—0·367	—0·716	0·221	0·622	0·343	—0·046	—0·080
14	0·017	—0·035	—0·445	0·085	—0·162	—0·579	0·238	0·661	0·135	—0·288	0·387

The constituents σ_1, MP_1, ψ_1 have the same values of ρ as MO_3, MK_3, K_1 respectively.

The constituents π_1, P_1, SO_1 have values of ρ equal to those of K_2, K_1, O_1, respectively but with opposite signs.

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TABLE III.—Values of cos $\rho\,(T - \overline{T})$ and sin $\rho\,(T - \overline{T})$ for semi-diurnal constituents.

Values of cos $\rho\,(T - \overline{T})$.

$T - \overline{T}$.	OQ_2	$2N_2$	N_2.	ν_2.	OP_2.	M_2.	λ_2.	L_2.	K_2.	KJ_2.
0	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000
1	0·442	0·636	0·794	0·812	0·896	0·911	0·974	0·981	0·999	0·966
2	−0·610	−0·191	0·261	0·319	0·606	0·659	0·898	0·923	0·998	0·865
3	−0·980	−0·879	−0·380	−0·294	0·190	0·290	0·775	0·830	0·995	0·706
4	−0·256	−0·927	−0·864	−0·796	−0·266	−0·131	0·612	0·704	0·991	0·498
5	0·755	−0·300	−0·992	−1·000	−0·666	−0·529	0·418	0·551	0·986	0·256
6	0·922	0·543	−0·711	−0·827	−0·928	−0·832	0·201	0·376	0·979	−0·004
7	0·059	0·994	−0·137	−0·344	−0·997	−0·987	−0·026	0·188	0·971	−0·263
8	−0·869	0·718	0·493	0·269	−0·859	−0·966	−0·251	−0·009	0·963	−0·504
9	−0·827	−0·080	0·921	0·780	−0·542	−0·772	−0·463	−0·205	0·953	−0·711
10	0·139	−0·820	0·968	1·000	−0·113	−0·441	−0·651	−0·393	0·942	−0·869
11	0·950	−0·963	0·617	0·842	0·340	−0·031	−0·806	−0·566	0·929	−0·968
12	0·700	−0·405	0·011	0·368	0·722	0·384	−0·919	−0·717	0·916	−1·000
13	−0·332	0·448	−0·599	−0·243	0·954	0·731	−0·984	−0·840	0·902	−0·964
14	−0·993	0·975	−0·962	−0·763	0·988	0·947	−0·999	−0·930	0·887	−0·862

Values of sin $\rho\,(T - \overline{T})$.

$T - \overline{T}$.	OQ_2	$2N_2$	N_2.	ν_2.	OP_2.	M_2.	λ_2.	L_2.	K_2.	KJ_2.
0	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
1	−0·897	−0·772	−0·608	−0·584	−0·444	−0·413	−0·226	−0·196	0·034	0·259
2	−0·792	−0·982	−0·965	−0·948	−0·796	−0·752	−0·440	−0·385	0·068	0·501
3	0·198	−0·476	−0·925	−0·956	−0·982	−0·957	−0·632	−0·558	0·102	0·709
4	0·967	0·375	−0·503	−0·605	−0·964	−0·991	−0·791	−0·710	0·136	0·867
5	0·656	0·954	0·126	−0·026	−0·746	−0·849	−0·909	−0·835	0·170	0·967
6	−0·387	0·838	0·703	0·562	−0·373	−0·555	−0·980	−0·927	0·204	1·000
7	−0·998	0·111	0·991	0·939	0·078	−0·162	−0·999	−0·982	0·238	0·965
8	−0·494	−0·696	0·870	0·963	0·512	0·260	−0·968	−1·000	0·272	0·863
9	0·562	−0·997	0·391	0·625	0·840	0·635	−0·886	−0·979	0·305	0·703
10	0·990	−0·572	−0·250	0·052	0·994	0·897	−0·759	−0·919	0·339	0·494
11	0·312	0·269	−0·787	−0·540	0·940	1·000	−0·592	−0·824	0·372	0·252
12	−0·714	0·915	−1·000	−0·930	0·692	0·923	−0·394	−0·697	0·405	−0·008
13	−0·943	0·894	−0·800	−0·970	0·299	0·683	−0·176	−0·543	0·438	−0·267
14	−0·119	0·222	−0·271	−0·645	−0·155	0·320	0·051	−0·368	0·471	−0·508

ρ of $MNS_2 = \rho$ of MN_4.
ρ of $\mu_2 = \rho$ of M_4.
ρ of $MKS_2 = \rho$ of MK_4.

ρ of $R_2 = \rho$ of K_1.
ρ of $T_2 = -\rho$ of K_1.

ρ of $MSN_2 = -\rho$ of λ_2.
ρ of $2SM_2 = -\rho$ of M_2.

TABLE IV.—Values of cos ρ (T — $\overline{\text{T}}$) and sin ρ (T — $\overline{\text{T}}$) for long period, third diurnal and compound constituents.

T — $\overline{\text{T}}$.	MO_3.	M_3.	MK_3.	MN_4.	M_4.	MK_4.	$2MN_6$.	M_6.	$2MK_6$.
				Values of cos ρ (T — $\overline{\text{T}}$).					
0	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000	1·000
1	0·641	0·803	0·918	0·472	0·660	0·925	0·066	0·290	0·685
2	—0·165	0·290	0·685	—0·554	—0·131	0·709	—0·991	—0·832	—0·063
3	—0·859	—0·337	0·339	—0·996	—0·832	0·387	—0·197	—0·772	—0·770
4	—0·946	—0·832	—0·062	—0·386	—0·965	0·006	0·965	0·384	—0·992
5	—0·363	—0·999	—0·454	0·631	—0·441	—0·375	0·325	0·995	—0·589
6	0·477	—0·772	—0·770	0·982	0·384	—0·700	—0·922	0·193	0·186
7	0·979	—0·242	—0·960	0·296	0·947	—0·920	—0·447	—0·883	0·844
8	0·788	0·384	—0·992	—0·702	0·865	—1·000	0·863	—0·705	0·969
9	0·039	0·859	—0·861	—0·959	0·193	—0·929	0·561	0·474	0·483
10	—0·737	0·995	—0·589	—0·203	—0·610	—0·718	—0·789	0·980	—0·308
11	—0·992	0·740	—0·219	0·767	—0·998	—0·399	—0·665	0·094	—0·904
12	—0·545	0·193	0·187	0·928	—0·705	—0·019	0·701	—0·925	—0·930
13	0·288	—0·430	0·561	0·109	0·068	0·364	0·758	—0·631	—0·370
14	0·917	—0·883	0·844	—0·825	0·795	0·691	—0·601	0·560	0·424
				Values of sin ρ (T — $\overline{\text{T}}$).					
0	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000	0·000
1	—0·763	—0·596	—0·397	—0·877	—0·752	—0·381	—0·998	—0·957	—0·729
2	—0·986	—0·957	—0·729	—0·833	—0·991	—0·705	—0·132	—0·555	—0·998
3	—0·511	—0·941	—0·941	0·095	—0·555	—0·922	0·980	0·635	—0·638
4	0·326	—0·555	—0·998	0·923	0·259	—1·000	0·262	0·923	0·125
5	0·932	0·050	—0·891	0·776	0·897	—0·927	—0·946	—0·100	0·809
6	0·879	0·635	—0·638	—0·190	0·923	—0·714	—0·387	—0·981	0·983
7	0·204	0·970	—0·280	—0·955	0·320	—0·393	0·895	—0·469	0·537
8	—0·616	0·923	0·125	—0·712	—0·502	—0·012	0·505	0·709	—0·248
9	—0·999	0·513	0·508	0·283	—0·981	0·370	—0·828	0·881	—0·876
10	—0·676	—0·100	0·809	0·979	—0·792	0·696	—0·614	—0·199	—0·952
11	0·126	—0·673	0·976	0·641	—0·063	0·917	0·747	—0·996	—0·427
12	0·839	—0·981	0·983	—0·373	0·709	1·000	0·713	—0·379	0·367
13	0·958	—0·903	0·828	—0·994	0·998	0·932	—0·652	0·775	0·929
14	0·399	—0·469	0·537	—0·565	0·606	0·722	—0·799	0·829	0·906

ρ of S*a* = ρ of K_1.
ρ of S*sa* = ρ of K_2.
ρ of M*m* = — ρ of λ_2.
ρ of MS*f* = — ρ of M_2.
ρ of M*f* = — ρ of MSK_2.

ρ of SO_3 = ρ of O_1.
ρ of SK_3 = ρ of K_1.

ρ of SN_4 = ρ of N_2.
ρ of MS_4 = ρ of M_2.
ρ of SK_4 = ρ of K_2.
ρ of MSN_6 = ρ of MN_4.
ρ of $2MS_6$ = ρ of M_4.
ρ of $2SM_6$ = ρ of M_2.
ρ of MSK_6 = ρ of MK_4.

TABLE V.—Values of $\rho\overline{T}$, cos $\rho\overline{T}$, sin $\rho\overline{T}$ for diurnal constituents.

$\overline{T}$.	$2Q_1$.	Q_1.	ρ_1.	O_1.	M_1.	χ_1.	K_1.	ϕ_1.	θ_1.	J_1.	OO_1.
Values of $\rho\overline{T}$.											
	°	°	°	°	°	°	°	°	°	°	°
15	307·54	143·52	169·75	339·49	177·14	205·04	14·78	44·35	184·53	210·76	50·08
44	254·13	108·99	185·92	323·85	183·61	265·44	43·37	130·11	181·29	258·23	122·89
74	149·21	36·02	165·41	282·83	177·88	315·52	72·94	218·81	190·36	319·75	223·04
103	95·80	1·49	181·58	267·18	184·35	15·92	101·52	304·57	187·12	7·22	295·86
133	350·88	288·53	161·08	226·17	178·63	66·00	131·09	33·27	196·19	68·74	36·01
163	245·97	215·56	140·57	185·16	172·91	116·07	160·66	121·98	205·25	130·25	136·17
Values of cos $\rho\overline{T}$.											
15	0·609	−0·804	−0·984	0·9366	−0·999	−0·906	0·9669	0·715	−0·997	−0·859	0·642
44	−0·274	−0·326	−0·995	0·8075	−0·998	−0·080	0·7270	−0·644	−1·000	−0·204	−0·543
74	−0·859	0·809	−0·968	0·2220	−0·999	0·714	0·2936	−0·779	−0·984	0·763	−0·731
103	−0·101	1·000	−1·000	−0·0492	−0·997	0·962	−0·1997	0·567	−0·992	0·992	0·436
133	0·987	0·318	−0·946	−0·6925	−1·000	0·407	−0·6573	0·836	−0·960	0·363	0·809
163	−0·407	−0·814	−0·772	−0·9959	−0·992	−0·439	−0·9436	−0·530	−0·904	−0·646	−0·721
Values of sin $\rho\overline{T}$.											
15	−0·793	0·595	0·178	−0·350	0·050	−0·423	0·2551	0·699	−0·079	−0·511	0·767
44	−0·962	0·946	−0·103	−0·590	−0·063	−0·997	0·6867	0·765	−0·023	−0·979	0·840
74	0·512	0·588	0·252	−0·975	0·037	−0·701	0·9560	−0·627	−0·180	−0·646	−0·682
103	0·995	0·026	−0·028	−0·999	−0·076	0·274	0·9797	−0·823	−0·124	0·126	−0·900
133	−0·159	−0·948	0·324	−0·721	0·024	0·914	0·7536	0·549	−0·279	0·932	0·588
163	−0·913	−0·582	0·635	−0·090	0·123	0·898	0·3312	0·848	−0·427	0·763	0·693

TABLE VI.—Values of $\rho\overline{T}$, cos $\rho\overline{T}$, sin $\rho\overline{T}$ for semi-diurnal constituents.

$\overline{T}$.	OQ_2.	$2N_2$.	N_2.	ν_2.	OP_2.	M_2.	λ_2.	L_2.	K_2.	KJ_2.
Values of $\rho\overline{T}$.										
	°	°	°	°	°	°	°	°	°	°
15	123·01	322·33	158·30	175·47	324·71	−5·72	164·02	190·25	29·57	225·54
44	72·83	297·49	152·35	130·71	280·47	7·21	145·14	222·07	86·74	301·60
74	318·85	222·15	108·96	121·65	209·88	−4·23	113·19	242·58	145·88	32·69
103	268·67	197·32	103·01	76·89	165·65	8·71	94·30	274·40	203·05	108·74
133	154·70	121·97	59·62	67·83	95·07	−2·74	62·36	294·90	262·18	199·83
163	40·72	46·63	16·22	58·77	24·48	−14·18	30·40	315·41	321·32	290·91

TABLE VI (continued).

T̄.	OQ_2.	$2N_2$.	N_2.	ν_2.	OP_2.	M_2.	λ_2.	L_2.	K_2.	KJ_2.
					Values of cos $\rho\bar{T}$.					
15	−0·545	0·791	−0·9291	−0·997	0·816	0·9950	−0·961	−0·984	0·8701	−0·700
44	0·295	0·463	−0·8858	−0·652	0·182	0·9921	−0·820	−0·742	0·0569	0·524
74	0·753	−0·741	−0·3249	−0·525	−0·867	0·9973	−0·394	−0·460	−0·8279	0·842
103	−0·023	−0·954	−0·2251	0·227	−0·969	0·9885	−0·075	0·077	−0·9202	−0·321
133	−0·904	−0·531	0·5057	0·378	−0·088	0·9989	0·463	0·421	−0·1361	−0·941
163	0·758	0·686	0·9601	0·520	0·910	0·9695	0·863	0·712	0·7807	0·357
					Values of sin $\rho\bar{T}$.					
15	0·839	−0·612	0·3698	−0·078	−0·578	−0·0996	0·276	−0·179	0·4937	−0·714
44	0·955	−0·886	0·4641	−0·758	−0·983	0·1255	0·572	−0·670	0·9984	−0·852
74	−0·658	−0·672	0·9457	−0·851	−0·498	−0·0738	0·919	−0·888	0·5608	0·540
103	−1·000	−0·299	0·9744	−0·974	0·248	0·1514	0·997	−0·997	−0·3915	0·947
133	0·427	0·847	0·8627	−0·926	0·996	−0·0478	0·886	−0·907	−0·9907	−0·339
163	0·652	0·728	0·2793	−0·854	0·414	−0·2450	0·506	−0·702	−0·6250	−0·934

TABLE VII.—Values of $\rho\bar{T}$, cos $\rho\bar{T}$ and sin $\rho\bar{T}$ for compound constituents.

T̄.	MO_3.	M_3.	MK_3.	MN_4.	M_4.	MK_4.	$2MN_6$.	M_6.	$2MK_6$.
					Values of $\rho\bar{T}$.				
15	333·77	171·42	9·06	152·58	−11·44	23·85	146·86	−17·17	18·12
44	331·06	190·82	50·58	159·57	14·42	93·95	166·78	21·64	101·17
74	278·60	173·65	68·71	104·73	−8·46	141·65	100·50	−12·69	137·41
103	275·89	193·06	110·23	111·72	17·42	211·75	120·42	26·12	220·46
133	223·43	175·89	128·35	56·88	− 5·48	259·44	54·14	− 8·22	256·70
163	170·97	158·72	146·48	2·04	−28·36	307·14	347·85	−42·55	292·95
					Values of cos $\rho\bar{T}$.				
15	0·897	−0·989	0·988	−0·888	0·980	0·914	−0·837	0·955	0·950
44	0·875	−0·982	0·635	−0·937	0·969	−0·070	−0·973	0·930	−0·194
74	0·150	−0·994	0·363	−0·254	0·989	−0·784	−0·182	0·976	−0·736
103	0·103	−0·974	−0·346	−0·370	0·954	−0·850	−0·506	0·898	−0·761
133	−0·726	−0·997	−0·620	0·547	0·995	−0·183	0·586	0·990	−0 230
163	−0·988	−0·932	−0·834	0·999	0·880	0·604	0·978	0·737	0·390
					Values of sin $\rho\bar{T}$.				
15	−0·442	0·149	0·158	0·461	−0·198	0·404	0·547	−0·295	0·311
44	−0·484	−0·188	0·773	0·349	0·249	0·998	0·229	0·369	0·981
74	−0·989	0·111	0·932	0·967	−0·148	0·620	0·983	−0·220	0·677
103	−0·995	−0·226	0·938	0·929	0·299	−0·526	0·862	0·440	−0·649
133	−0·688	0·072	0·784	0·837	−0·096	−0·983	0·810	−0·143	−0·973
163	0·157	0·363	0·552	0·035	−0·475	−0·797	−0·211	−0·676	−0·921

Table VIII.—Values of a, b, D, M for diurnal constituents.

—	$2Q_1$.	σ_1.	Q_1.	ρ_1.	O_1.	MP_1.	M_1.	χ_1.	π_1.
a_1	35·375	35·267	34·413	34·260	33·158	32·948	31·638	31·409	30·304
b_1	—22·222	—22·310	—22·774	—22·830	—23·095	—23·125	—23·181	—23·177	—23·081
D_0	1·036	0·054	—0·896	0·444	0·618	—1·746	0·532	5·590	27·822
D_1	—0·832	1·282	3·314	0·010	—2·862	3·488	28·146	24·730	1·422
D_2	2·562	—0·666	—6·264	—1·032	31·574	30·132	0·484	—1·828	—0·354
D_3	—3·332	0·852	30·420	29·964	2·342	—0·832	1·500	2·044	0·206
D_4	30·106	31·126	3·516	—0·592	—1·220	0·092	—1·122	—1·392	—0·136
D_a	0·858	1·126	0·486	—0·678	—1·482	2·024	29·660	30·672	9·222
D_b	0·030	—1·016	—4·218	—1·430	28·964	30·476	0·634	—4·872	—4·716
D_c	—3·174	—1·400	27·638	29·624	2·692	—3·462	—2·086	1·208	2·680
D_d	27·516	28·564	4·198	—0·386	—1·484	1·036	—0·974	—2·810	—2·152
M_0	—0·090	0·622	0·366	—11·330	0·457	0·372	—11·970	1·316	—0·353
M_1	0·026	10·836	—1·630	—0·882	11·272	11·216	—0·028	—3·338	0·928
M_2	2·324	—0·688	—6·854	0·424	—0·464	0·274	0·010	—6·042	6·798
M_3	6·070	0·474	1·690	—0·390	0·323	—0·284	—0·014	0·536	—0·392
M_a	—1·868	—13·194	2·474	3·406	—14·020	15·128	0·034	—1·090	—0·445
M_b	—3·938	—0·370	11·054	—2·062	0·052	—0·844	—0·268	—12·236	—12·098
M_c	—8·668	1·054	—1·206	1·620	0·446	0·794	0·346	1·638	0·586

—	P_1.	S_1.	K_1.	ψ_1.	ϕ_1.	θ_1.	J_1.	SO_1.	OO_1.
a_1	30·176	30·033	29·895	29·758	29·620	28·256	27·993	26·236	25·919
b_1	—23·064	—23·045	—23·025	—23·004	—22·983	—22·704	—22·638	—22·126	—22·018
D_0	28·708	29·000	28·708	27·822	26·370	0·264	—3·292	0·618	2·514
D_1	0·354	—	0·354	1·422	3·146	28·282	29·162	—2·862	—7·298
D_2	—0·104	—	—0·104	—0·354	—0·740	0·664	4·290	31·574	30·206
D_3	0·054	—	0·054	0·206	0·460	1·444	0·476	2·342	6·940
D_4	—0·044	—	—0·044	—0·136	—0·298	—1·094	—0·584	—1·220	—2·838
D_a	4·624	—	—4·624	—9·222	—13·524	—29·532	—26·756	1·482	3·612
D_b	—2·380	—	2·380	4·716	6·624	—1·014	—7·322	—28·964	—25·198
D_c	1·360	—	—1·360	—2·680	—3·712	2·300	5·528	—2·692	—9·842
D_d	—1·088	—	1·088	2·152	3·032	0·854	—1·006	1·484	4·052
M_0	0·374	12·000	0·374	—0·353	0·330	—11·674	0·818	0·457	—0·216
M_1	11·397	—	11·397	0·928	—0·672	—0·436	—2·444	11·272	0·414
M_2	—0·141	—	—0·141	6·798	0·794	0·150	—6·520	—0·464	0·432
M_3	0·066	—	0·066	—0·392	8·142	—0·122	1·166	0·323	7·764
M_a	—14·677	—	14·677	0·445	2·550	—3·436	—1·764	14·020	2·304
M_b	0·468	—	—0·468	12·098	0·958	1·608	—11·736	—0·052	1·592
M_c	—0·182	—	0·182	—0·586	8·622	—1·008	1·450	—0·446	8·940

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Table IX.—Values of *a*, *b*, D, M for semi-diurnal constituents.

—	OQ_2.	MNS_2.	$2N_2$.	μ_2.	N_2.	ν_2.	OP_2.	M_2.	MKS_2.
a_2	35·932	35·942	35·905	35·882	35·668	35·621	35·316	35·237	35·156
b_2	—31·250	—31·393	—32·151	—32·254	—32·867	—32·949	—33·343	—33·411	—33·479
D_0	—0·800	0·120	0·498	—0·540	—0·160	1·246	1·650	—0·534	—2·956
D_1	4·186	2·076	0·308	2·606	1·514	—2·002	—5·358	0·116	7·120
D_2	—2·240	0·456	0·826	—2·698	—3·500	2·498	31·250	31·190	28·456
D_3	2·596	—0·200	—1·132	3·754	30·400	28·880	4·476	0·564	—1·818
D_4	—6·592	—1·906	30·938	30·754	1·056	—2·348	—2·014	—0·512	0·548
D_5	27·570	28·376	4·022	0·890	1·668	2·308	—1·184	—2·048	—2·440
D_a	2·422	2·112	1·002	1·330	—0·148	—1·394	—2·710	0·110	4·238
D_b	—0·772	—0·454	—0·508	—1·746	—2·802	0·684	27·334	30·028	30·258
D_c	4·172	3·246	—2·342	0·206	28·982	29·894	6·196	—0·566	—5·898
D_d	1·150	4·564	28·302	28·436	1·506	—2·506	—2·796	—0·180	2·084
D_e	29·374	30·496	3·686	—0·558	3·030	5·330	2·050	—0·932	—3·582
M_0	0·668	—1·806	—0·572	11·534	—1·798	—2·098	—0·032	11·883	—0·738
M_1	—1·262	—10·284	2·834	0·418	—10·539	—9·632	0·368	0·106	1·598
M_2	—1·034	1·470	6·344	—0·166	1·162	—0·358	7·124	—0·043	6·304
M_3	—6·556	—1·038	—2·204	0·182	—0·796	0·530	—0·932	0·047	0·262
M_a	1·878	13·320	—3·808	—0·130	14·286	—15·900	—1·276	—0·068	—0·350
M_b	3·170	—1·024	—10·358	1·040	—1·471	2·470	—11·392	0·534	12·618
M_c	9·062	—0·428	2·096	—1·342	0·112	—1·582	0·198	—0·689	—0·944

—	λ_2.	L_2.	T_2.	S_2.	R_2.	K_2.	MSN_2.	KJ_2.	$2SM_2$.
a_2	34·713	34·620	34·005	33·941	33·877	33·812	33·005	32·853	32·070
b_2	—33·737	—33·775	—33·933	—33·941	—33·949	—33·958	—33·960	—33·941	—33·823
D_0	—1·442	2·786	28·708	29·000	28·708	27·822	—1·442	—4·708	—0·534
D_1	28·948	26·814	0·354	—	0·354	1·422	28·948	28·636	0·116
D_2	2·094	—0·800	—0·104	—	—0·104	—0·354	2·094	6·856	31·190
D_3	1·066	1·820	0·054	—	0·054	0·206	1·066	—0·188	0·564
D_4	—0·904	—1·290	—0·044	—	—0·044	—0·136	—0·904	—0·238	—0·512
D_5	0·478	0·928	0·028	—	0·028	0·106	0·478	—0·284	—2·048
D_a	28·506	30·394	4·624	—	—4·624	—9·222	—28·506	—24·594	—0·110
D_b	3·680	—2·148	—2·380	—	2·380	4·716	—3·680	—11·088	—30·028
D_c	—3·734	—0·482	1·360	—	—1·360	—2·680	3·734	7·172	0·566
D_d	—0·038	—1·884	—1·088	—	1·088	2·152	0·038	—1·964	0·180
D_e	—2·806	—0·908	0·748	—	—0·748	—1·474	2·806	4·696	0·932
M_0	—1·848	—1·952	0·374	12·000	0·374	—0·353	—1·848	—0·478	11·883
M_1	—10·500	—10·184	11·397	—	11·397	0·928	—10·500	1·028	0·106
M_2	0·742	0·222	—0·141	—	—0·141	6·798	0·742	—1·728	—0·043
M_3	—0·444	0·008	0·066	—	0·066	—0·392	—0·444	—7·370	0·047
M_a	15·060	—15·610	—14·677	—	14·677	0·445	—15·060	—2·112	0·068
M_b	—1·872	2·212	0·468	—	—0·468	12·098	1·872	—2·426	—0·534
M_c	0·648	—1·146	—0·182	—	0·182	—0·586	—0·648	—8·652	0·689

TABLE X.—Values of x, D, M for long-period constituents.

—	A_0.	Sa.	Ssa.	Mm.	MSf.	Mf.
x_0	33·00	30·00	30·00	29·87	29·56	29·49
D_0	29·000	28·708	27·822	—1·442	—0·534	1·650
D_1	—	0·354	1·422	28·948	0·116	—5·358
D_2	—	—0·104	—0·354	2·094	31·190	31·250
D_a	—	—4·624	—9·222	—28·506	—0·110	2·710
D_b	—	2·380	4·716	—3·680	—30·028	—27·334
M_0	12·000	0·374	—0·353	—1·848	11·883	—0·034
M_1	—	11·397	0·928	—10·500	0·106	0·370
M_2	—	—0·141	6·798	0·742	—0·043	7·124
M_a	—	14·677	0·445	—15·060	0·068	1·274
M_b	—	—0·468	12·098	1·872	—0·534	11·394

TABLE XI.—Values of x, D, M for third-diurnal constituents.

—	MO_3.	M_3.	SO_3.	MK_3.	SK_3.
x_3	31·73	31·56	31·10	30·98	28·86
D_0	0·054	0·538	0·618	—1·746	28·708
D_2	—0·666	—0·656	31·574	30·132	—0·104
D_3	0·852	29·874	2·342	—0·832	0·054
D_4	31·126	—0·810	—1·220	0·092	—0·044
D_b	—1·016	—1·208	28·964	30·476	2·380
D_c	—1·400	29·682	2·692	—3·462	—1·360
D_d	28·564	—0·642	—1·484	1·036	1·088
M_0	0·622	—11·736	0·457	0·372	0·374
M_1	10·836	—0·238	11·272	11·216	11·397
M_a	—13·914	0·100	—14·020	15·128	14·677

TABLE XII.—Values of a, b, D, M for quarter-diurnal constituents.

—	MN_4.	M_4.	SN_4.	MS_4.	MK_4.	S_4.	SK_4.
a_4	16·269	16·380	16·402	16·344	16·327	16·000	15·960
b_4	—25·463	—26·180	—26·703	—27·183	—27·244	—27·713	—27·731
D_0	0·120	—0·540	—0·160	—0·534	—2·956	29·000	27·822
D_2	0·456	—2·698	—3·500	31·190	28·456	—	—0·354
D_3	—0·200	3·754	30·400	0·564	—1·818	—	0·206
D_4	—1·906	30·754	1·056	—0·512	0·548	—	—0·136
D_5	28·376	0·890	1·668	—2·048	—2·440	—	0·106
D_b	—0·454	—1·746	—2·802	30·028	30·258	—	4·716
D_c	3·246	0·206	28·892	—0·566	—5·898	—	—2·680
D_d	4·564	28·436	1·506	—0·180	2·084	—	2·152
D_e	30·496	—0·558	3·030	—0·932	—3·582	—	—1·474
M_0	—1·806	11·534	—1·798	11·883	—0·738	12·000	—0·353
M_1	—10·284	0·418	—10·539	0·106	1·598	—	0·928
M_2	1·470	—0·166	1·162	—0·043	6·304	—	6·798
M_a	13·320	—0·130	14·286	—0·068	—0·350	—	0·445
M_b	—1·024	1·040	—1·471	0·534	12·618	—	12·098

TABLE XIII.—Values of x, D, M for sixth-diurnal constituents.

—	$2MN_6$.	M_6.	MSN_6.	$2MS_6$.	$2MK_6$.	$2SM_6$.	MSK_6.
x_6	32·04	33·11	33·85	34·70	34·80	35·64	35·70
D_0	0·254	—0·556	0·120	—0·540	—1·670	—0·534	—2·956
D_2	2·626	—2·448	0·456	—2·698	—6·704	31·190	28·456
D_4	—1·224	—1·482	—1·906	30·754	28·128	—0·512	0·548
D_5	—1·052	1·322	28·376	0·890	—1·280	—2·048	—2·440
D_6	0·776	30·022	4·770	0·948	2·462	—2·548	—1·742
D_7	28·236	—0·364	—2·310	—0·230	—0·892	—0·162	0·198
D_b	—0·104	0·184	—0·454	—1·746	—3·396	30·028	30·258
D_d	1·232	—0·552	4·564	28·436	26·644	—0·180	2·084
D_e	—2·644	4·772	30·496	—0·558	—3·684	—0·932	—3·582
D_f	—4·194	29·312	5·368	—1·090	1·586	—0·318	1·272
D_g	29·544	—0·624	—0·540	—1·472	—3·130	—2·914	—4·414
M_0	—1·868	10·972	—1·806	11·534	—1·162	11·883	—0·738
M_1	—9·730	0·908	—10·284	0·418	2·362	0·106	1·598
M_2	1·658	—0·364	1·470	—0·166	5·674	—0·043	6·304
M_a	12·208	—0·158	13·320	—0·130	—1·076	—0·068	—0·350
M_b	—0·566	1·490	—1·024	1·040	12·932	0·534	12·618

Instructions to Computers.

11. *Preliminary Remarks.*

The *Data* are supposed to consist of hourly heights for about 360 days. It is sufficient that these should be tabulated to the nearest tenth of a foot. Notes should be made concerning the kind of time used and the datum of the observations. The best practice is to record observations in standard time with zero hour at midnight. The observations should be written on a standard form; otherwise certain stencils required for "the daily processes" will have to be cut to suit the form used.

11.1. *Breaks in the Record* should only be occasional and should not extend as a rule over more than two or three days at a time. If only a few hours' observations are missing, they can be interpolated by means of a graph of the hourly heights on each side of the gap. A better method is to graph the heights at intervals of 25 hours, as these are more nearly constant than any other set of heights and, further, the gap is more easily bridged than by the first method. This second method can be used even if the break in the records covers three days. If, however, the breaks in the record cover more than three days at a time, or if the first two methods do not appear to be satisfactory, proceed with the calculations for "the daily processes," which yield certain functions X, Y, which are tabulated in 12 columns with 29 or 30 entries in the column, as explained later. Then the interpolations may be made preferably along a row, with checks along the diagonals, and a final check down the column. Thus, if 10 days' observations are lacking in a column of X, we should have three ways (row and two diagonals) of filling in each entry, with a final test down the column.

11.2. *Day Numbers.*—It is convenient to number the days, taking for zero day the middle day of the observations chosen for analysis. As there is usually a little choice available, the first day of a calendar month may be taken. Certain day numbers are important, and the days should be counted backwards and forwards and the day numbers,

$$T = -177, -148, -118, -88, -59, -29, 0, 30, 59, 89, 118, 148, 177,$$

should be specially marked. It is unnecessary to number each day.

11.3. *Forms for X, Y.*—We propose to combine the hourly heights on each day in special ways, and the numerical results of each combination (X_p or Y_p) are to be entered on special forms, which must be prepared first. It is important to rule these forms correctly, a specimen heading being shown below.

	−148	−118		−59		0		59		118	148
−177			−88		−29		30		89		

An example for Vancouver is given in Table XXX. There are 12 columns, each about 0·4 inch wide, and space is left for 29 rows beneath the horizontal and partly dotted line. The first entry in the form will be for day —177 and the results for successive days will be entered in the column until day —148 is reached, for which the numerical value of X_p or Y_p will be placed *above the dotted line*; the results for the next 29 days are to be placed in the same column. Thus we see the importance of having days —177, —148, ..., specially marked.

It will be necessary to prepare nine such forms, one each for X_0, X_1, Y_1, X_2, Y_2, X_3, X_4, Y_4, X_6. In the Tidal Institute two forms are placed together on the standard sheet used for the tabulation of hourly heights.

11.4. *Stencils.*—The combinations of hourly heights for X and Y are given in a compact form in Table XIV. If ζ_0, ζ_1, ζ_2, ... are heights of tide at hours 0, 1, 2, ..., then, written at length, the combination for X_2 is

$$(\zeta_0 + 2\zeta_2 + \zeta_4) - (2\zeta_6 + 4\zeta_8 + 2\zeta_{10}) + (2\zeta_{12} + 4\zeta_{14} + 2\zeta_{16}) - (2\zeta_{18} + 4\zeta_{20} + 2\zeta_{22}) + (\zeta_{24} + 2\zeta_{26} + \zeta_{28}),$$

and this should serve to explain the table. The formula for X_2 is exhibited in another way in Table XXIX as a stencil, and it is necessary for the computer to cut nine stencils, once for all, to suit his standard form for hourly heights. Thus for X_2 a blank form for hourly heights is taken and holes cut for hours 0, 2, 4, 6, ..., 26, 28. Adjacent to each opening, above or below, the multipliers given in the table should be written. Negative multipliers are preferably written in red ink. A hole is cut at the left-hand side of the stencil so as to reveal the date or the day number, or both together. It is important to cut this on the first line of the stencil, *except for* X_1; in the latter case only a few holes appear on the first line and the appropriate line for the day number to appear is the second one. Follow the instructions in Table XIV.

11.5. *Tables of Multipliers.*—It is also necessary to prepare, once for all, a manuscript copy of Tables XV, XVI to suit the spacing of the form referred to in § 11.3. From Table XV we have 29 entries in each column, but only half of the table is printed and the instructions must be followed. It is also necessary to write out the table with a little space left between each column so that the table can be folded.

Table XVI should be copied twice, once with a spacing for the 12 columns of $\overline{T}$ identical with that of the form of § 11.3 and once to suit columns about 50 per cent. wider. Space must be left between the *rows* for folding.

In both cases negative multipliers are preferably written in red ink.

11.6. *Characteristic Features of the Method.*—The computer is now ready to commence calculating, but it is desirable first to explain the general character of the computations. At each stage it is necessary to combine a number of quantities, each of which has a numerical multiplier with positive or negative sign. Thus a stencil placed over hourly heights will reveal the quantities to be multiplied by the factors written on the stencil, and the sum of the products is required. Again, it will be necessary to take 29 numbers and to use the 29 multipliers in a column of Table XV; in this case the manuscript table can be folded to bring the multipliers alongside the multiplicands, and again the products have to be summed. Similarly 12 numerical quantities may have to be combined by use of the multipliers in a row of Table XVI.

The multiplications are all simple in character; it is advisable to sum the negative products separately from the positive products, but the separate sums are not required. The process is simplified if the quantities to be combined are all positive, and also if a calculating machine is used. We shall deal next with these matters, and then with the principles of checking.

11.7. *Calculating Machines* are not strictly essential to the method, but if an adding machine of the Comptometer type is available, then it is an easy matter to perform the simple multiplications of the processes, either mentally or on the machine, and to sum continuously. The negative contributions may be summed first and the sum subtracted from zero; for instance, if the negative contributions totalled 769, this could be replaced by 99999231 and positive contributions could be added as usual. Thus no separate writing of positive and negative contributions is necessary.

With the electrically driven Monroe machine the question of sign causes no difficulty, for the machine adds with a touch on one bar and subtracts with a touch on the other bar.

Both these machines have keyboards for setting, and are superior to lever operated machines for this class of work.

11.8. *A Datum* may be used to give the advantage of having to deal only with positive quantities. Wherever required, a suitable value for datum will be suggested. A datum yields nothing to the results of any process, for the sum of the multipliers in any column or row has been made zero. The only exception is when the quantities treated have simply to be summed, in which case it is necessary to subtract 29 times, or 12 times, the datum according to the number of quantities involved.

11.9. *Checks* of one sort or another are highly desirable, and are of three kinds :—

(*a*) If a large number of values of a function are available and are given at regular intervals of time, then considerations of smoothness are sufficient. The values of X_p, Y_p are specially suitable for these tests. Any discordant values will have to be tested by repetition.

(*b*) Summation methods may be used; thus, if quantities A, B, C ... have to be multiplied by a, b, c ... respectively, and the products summed to give S, and by a', b', c' ... to give S′, and by a'', b'', c'' . . to give S″, then obviously if A, B, C ... are multiplied by $a + a' + a''$, $b + b' + b''$, ... respectively and the the products summed, the result should equal $S + S' + S''$. As a rule it is not advisable to combine more than five multipliers for this test.

(*c*) Repetition is, of course, most satisfactory, but requires most time. It is inadvisable for the computer to carry out repetition tests himself; but if no other person is available, then he should leave the tests until the next day, or even later. It is possible for a very careless computer to use the wrong set of multipliers, and this contingency requires caution to be exercised even when repetition tests only are made.

12. *Daily Processes.*

(*a*) Enter the following datum values* on the forms for X, Y referred to in § 11.3.

X_0	:	0	X_2, Y_2	:	500
X_1, Y_1	:	200	X_4, Y_4	:	30
X_3	:	20	X_6	:	30

(*b*) Compute the values of

$$X_0,\ X_1,\ Y_1,\ X_2,\ Y_2,\ X_3,\ X_4,\ Y_4,\ X_6,$$

adding the appropriate datum as in (*a*).

Use the stencils one at a time until the whole of the corresponding values for the function are computed. It is inadvisable to use two stencils alternately.

(*c*) Enter the values of X, Y, plus datum, on the forms already prepared, to the nearest foot only.

(*d*) Check the results by smoothness tests, either by eye or graphically, and recompute any doubtful values.

13. *Monthly Processes.**

(*a*) Prepare nine forms each with 13 columns each about 0·6-inch wide. In the first columns of each form enter the symbols and datum values contained in the nine sections of the following table. An example for Vancouver is given in Table XXXI.

X_{00}	$X_{10}+2000$	$Y_{10}+2000$	$X_{20}+5000$	$Y_{20}+5000$	$X_{30}+200$	$X_{40}+300$	$Y_{40}+300$	$X_{62}+300$
$X_{01}+100$	$X_{11}+100$	$Y_{11}+100$	$X_{21}+300$	$Y_{21}+300$	$X_{32}+200$	$X_{42}+300$	$Y_{42}+300$	$X_{6b}+300$
$X_{0a}+100$	$X_{1a}+100$	$Y_{1a}+100$	$X_{2a}+300$	$Y_{2a}+300$	$X_{3b}+200$	$X_{4b}+300$	$Y_{4b}+300$	$X_{64}+300$
$X_{02}+100$	$X_{12}+2000$	$Y_{12}+2000$	$X_{22}+10000$	$Y_{22}+10000$	$X_{33}+200$	$X_{43}+300$	$Y_{43}+300$	$X_{6d}+300$
$X_{0b}+100$	$X_{1b}+2000$	$Y_{1b}+2000$	$X_{2b}+10000$	$Y_{2b}+10000$	$X_{3c}+200$	$X_{4c}+300$	$Y_{4c}+300$	$X_{65}+300$
	$X_{13}+400$	$Y_{13}+400$	$X_{23}+2000$	$Y_{23}+2000$	$X_{34}+200$	$X_{44}+300$	$Y_{44}+300$	$X_{6e}+300$
	$X_{1c}+400$	$Y_{1c}+400$	$X_{2c}+2000$	$Y_{2c}+2000$	$X_{3d}+200$	$X_{4d}+300$	$Y_{4d}+300$	$X_{66}+300$
	$X_{14}+100$	$Y_{14}+100$	$X_{24}+1000$	$Y_{24}+1000$		$X_{45}+300$	$Y_{45}+300$	$X_{6f}+300$
	$X_{1d}+100$	$Y_{1d}+100$	$X_{2d}+1000$	$Y_{2d}+1000$		$X_{4e}+300$	$Y_{4e}+300$	$X_{67}+300$
			$X_{25}+100$	$Y_{25}+100$				$X_{6g}+300$
			$X_{2e}+100$	$Y_{2e}+100$				

The datum suggested is larger than may be necessary; it is suitable for a range of 40 feet of semi-diurnal tide and of 10 feet of diurnal tide. The computer may take smaller datum values in proportion to the range of tide.

(*b*) Taking functions arising from X_1 as an example, take the sum of the 29 values in a column, subtract 29 times the datum used with X_1 (*i.e.*, 29×200) and add 2000. The result is called $X_{10} + 2000$, and from the 12 columns of values of X_1, ignoring the values above the dotted line, we get 12 values, which must be entered on the appropriate form.

(*c*) Taking the manuscript copy of Table XV, fold it so that the multipliers with

* When the computer is familiar with the monthly and annual processes he may read § 15, where an alternative and better procedure is advocated.

suffix 1 can be placed alongside the 29 values of X_1 + datum in a column of values of that function. Take the sum of the products of the terms and add 100. From 12 such operations 12 values of $X_{11} + 100$ are to be obtained and entered on the second row of the appropriate form.

(*d*) Refold the table of multipliers so as to place those with suffix *a* alongside the column of 29 values of X_1 + datum. Sum the products and add 100, so obtaining values of $X_{1a} + 100$.

(*e*) The calculation of the remaining functions is carried out in precisely the same way. The second suffix denotes the multipliers used and the first suffix denotes the function X_p with which they are used.

It is necessary to remember that if the second suffix is 0, then 29 times the datum for X_p must be subtracted before adding the new datum given in the table above.

(*f*) These operations can be checked by the summation type of check. Thus the sum $X_{00} + X_{01} + X_{0a} + X_{02} + X_{0b}$ obtained for any month should equal the result of applying the sum of the multipliers with suffixes 0, 1, *a*, 2, *b* to the values of X in that month. Do not test more than four or five functions together.

14. *Annual Processes.*

(*a*) Prepare six blank forms similar to those of Tables XVII to XXII, but with an extra row under "correction terms," an extra column for the principal terms, and columns for "sum" "[R cos δ] or [R sin δ]," "R cos δ or R sin δ," and "constituent." The following illustrates the heading of the form corresponding to Table XIX and shows the only entries to be made at this stage.

Principal Term.		Correction terms arising from							Sum.	Divisor.	[R cos δ]	Princ. Const.	R cos δ.	Const.
Symbol.	Value.	A_{101}.	B_{10a}.	A_{110}.	A_{112}.	A_{121}.	A_{123}.	A_{132}.						
A_{100}										10452		S_1		S_1
A_{101}										9782		K_1*		K_1
B_{10a}										9700		K_1*		P_1
...										...		...		

An illustration of the form corresponding to Table XXI is given in the example for Vancouver, Table XXXIV.

The columns for the symbols and the correction terms should be about 0·4 inch wide and the rest 0·6 inch wide.

(*b*) Prepare subsidiary forms each with five columns 0·6 inch wide, headed

Suffix, X, Y, A, B.

Under "suffix" write the triple suffixes of the "Principal Functions" of Tables XIX to XXII, and the suffixes commencing with 4 in Tables XVII and XVIII.

(*c*) To compute X_{100} sum the 12 values of $X_{10} + 1000$ and subtract 12 times the datum (*i.e.*, 12×1000). Enter the result on the subsidiary form under X and in the same row as the suffix 100.

Similarly compute Y_{100}, entered under Y; and all functions in which the last suffix is zero, remembering always to subtract 12 times the datum.

(*d*) Using the manuscript copy of Table XVI, fold it so that the 12 multipliers for suffix 1 can be placed underneath the 12 values of $X_{10} + 1000$. Sum the products in the usual way and enter on the subsidiary form against the symbol 101. It should be clear to the computer that the first two figures in the triple suffix indicate the function used and the third figure indicates the multiplier used. An example is given in Table XXXII.

Similarly compute all the quantities for which provision is made on the subsidiary forms.

(*e*) Similarly compute the values of the "principal terms" on the forms corresponding to Tables XVII and XVIII (first suffix = 0, 3 or 6) and enter the results direct on those forms.

(*f*) Check by repetition (see § 11.9).

(*g*) Finally compute A and B on the subsidiary form from

$$A = X + Y, \qquad B = X - Y.$$

Check by repetition and enter the results on the forms corresponding to Tables XVII to XXII, special attention being paid to the signs and to the placing of A and B.

15. *Alternative Procedure.*

An alternative procedure, which has much to recommend it, is to perform the annual process before the monthly process, that is, to take the 12 values in each row of the table of X_p, apply the multipliers m_0, m_1, m_a, ..., and so to obtain functions denoted by $X_{p\cdot 0}$, $X_{p\cdot 1}$, $X_{p\cdot a}$ .... The functions required, plus suitable datum values, are given below.*

$X_{0\cdot 0}$	$X_{1\cdot 0}+500$	$Y_{1\cdot 0}+500$	$X_{2\cdot 0}+7000$	$Y_{2\cdot 0}+7000$	$X_{3\cdot 0}+200$	$X_{4\cdot 0}+200$	$Y_{4\cdot 0}+200$	$X_{6\cdot 0}+200$
$X_{0\cdot 1}+300$	$X_{1\cdot 1}+2000$	$Y_{1\cdot 1}+2000$	$X_{2\cdot 1}+2000$	$Y_{2\cdot 1}+2000$	$X_{3\cdot 1}+200$	$X_{4\cdot 1}+200$	$Y_{4\cdot 1}+200$	$X_{6\cdot 1}+200$
$X_{0\cdot a}+300$	$X_{1\cdot a}+2000$	$Y_{1\cdot a}+2000$	$X_{2\cdot a}+2000$	$Y_{2\cdot a}+2000$	$X_{3\cdot a}+200$	$X_{4\cdot a}+200$	$Y_{2\cdot a}+200$	$X_{6\cdot a}+200$
$X_{0\cdot 2}+300$	$X_{1\cdot 2}+500$	$Y_{1\cdot 2}+500$	$X_{2\cdot 2}+1000$	$Y_{2\cdot 2}+1000$		$X_{4\cdot 2}+200$	$Y_{4\cdot 2}+200$	$X_{6\cdot 2}+200$
$X_{0\cdot b}+300$	$X_{1\cdot b}+500$	$Y_{1\cdot b}+500$	$X_{2\cdot b}+1000$	$Y_{2\cdot b}+1000$		$X_{4\cdot b}+200$	$Y_{4\cdot b}+200$	$X_{6\cdot b}+200$
	$X_{1\cdot 3}+100$	$Y_{1\cdot 3}+100$	$X_{2\cdot 3}+100$	$Y_{2\cdot 3}+100$				
	$X_{1\cdot c}+100$	$Y_{1\cdot c}+100$	$X_{2\cdot c}+100$	$Y_{2\cdot c}+100$				

An example is given in Table XXXIII.

* If for any reason a full analysis is not required, the functions with suffixes 2.3, 2.*c*, 4.2, 4.*b*, 6.2, 6.*b* may be omitted, but there is little to be gained by doing so.

The numerical values are entered in columns headed as above, and there will be 29 values in each column. The daily multipliers (Table XIV) can then be applied to give the quantities with triple suffixes required for Tables XVII to XXII; the instructions of §§ 14*a*, *b*, *e*, *f*, *g* should be followed.

The columns of 29 values may often be tested by considerations of smoothness, and finally by the summation tests. This alternative procedure is much quicker than the "direct methods" of §§ 13 and 14, and the checks are easier to apply.

16. *Calculation of* [$R \cos \delta$] *and* [$R \sin \delta$].

The six forms prepared under Instruction 14 (*a*) are now to be completed, after carrying out Instructions 14 (*e*) and 14 (*g*).

(*a*) Under the heading "correction terms" in each form are symbols for certain functions, and space has been left for the numerical values to be written directly underneath the symbols. These values, of course, are taken from the column of values of the "principal term."

(*b*) Referring to Table XIX, the numerical value of A_{101} has to be multiplied by factors given in the same column, viz., $-$ 0·033, ..., ..., 0·012, ..., $-$0·006, ... The products must be accurately placed on the form; if there is no multiplier fill in the space with two or three dots; if the product is negligible write 0 and do not use dots. Fill in the whole of the correction terms in this manner. An example is given in Table XXXIV.

(*c*) Verify that

(i) The correct table of multipliers has been used;
(ii) The spacing of the terms is correct in the rows and columns;
(iii) The signs are correct;
(iv) The approximate magnitudes are correct.

If these tests are carried out as suggested, the computer may check his own work. To check approximate magnitudes consider 0·033 as 1/30, 0·012 as 1/80 or 1/100, and so on. Last figure accuracy is the very least important matter and need not be tested.

(*d*) Sum the "principal term" and the correction terms in the same row, enter under "sum," divide by the divisor and enter the quotient under [R cos δ] or [R sin δ] as the case may be.

This section of the work must be checked by repetition (§ 11.9).

17. *Calculation of R* cos δ *and R* sin δ.

Referring to Table XXI, notice that against S_2 only one value of [R cos δ] is recorded; for T_2*, K_2, L_2** there are two, two and four values respectively. Let these be called in order A, B if there are only two, or A, B, C, D if there are four. The asterisk (*) means that while T_2 is the principal constituent, the quantities [R cos δ], [R sin δ] contain contributions from one other constituent, in this case R_2. The double asterisk means that two other constituents are involved. The two or four values A, B, C, D are combined by the formulæ of Table XXIII ; an example is given in Table XXXIV.

(*a*) Again referring to the form corresponding to Table XXI, combine the two values of [R cos δ] for T_2* by the formula

$$0\cdot50A + 0\cdot50B, \qquad 0\cdot50A - 0\cdot50B,$$

as indicated in Table XXIII. The results are the values of R cos for T_2 and R_2 respectively ; they should be entered under R cos δ and the symbols T_2, R_2 placed alongside them in the last column of the form.

(*b*) Combine the four values of [R cos δ] for L_2**, so obtaining one value of R cos δ for L_2, two values for λ_2, and one value for MSN_2, entering the results in the last two columns of the form.

(*c*) Complete the use of Table XXIII and then, for constituents whose symbols are not asterisked, copy under R cos δ the values of [R cos δ]. Similarly copy values of [R sin δ] under R sin δ for these constituents.

(*d*) Check by repetition.

18. *Examination of Results.*

If two or more values of R cos δ or R sin δ are given ostensibly for a single constituent, they should be in fairly close agreement. It is difficult, however, to specify how nearly the values should agree, but the following results for Vancouver may be useful as a guide (see also § 8).

$$OO_1\begin{cases}R\cos\delta = 0\cdot097,\ 0\cdot070,\ 0\cdot092,\ 0\cdot060\\ R\sin\delta = 0\cdot001,\ 0\cdot036,\ -0\cdot084,\ -0\cdot036\end{cases}$$

$$Q_1\begin{cases}R\cos\delta = 0\cdot096,\ 0\cdot106,\ 0\cdot150,\ 0\cdot074\\ R\sin\delta = 0\cdot177,\ 0\cdot160,\ 0\cdot187,\ 0\cdot158\end{cases}$$

$$2Q_1\begin{cases}R\cos\delta = -0\cdot012,\ -0\cdot022,\ 0\cdot001,\ 0\cdot023\\ R\sin\delta = -0\cdot034,\ -0\cdot008,\ -0\cdot023,\ -0\cdot014\end{cases}$$

$$K_2\begin{cases}R\cos\delta = 0\cdot101,\ 0\cdot103.\\ R\sin\delta = 0\cdot144,\ 0\cdot126.\end{cases}$$

$$\lambda_2\begin{cases}R\cos\delta = 0\cdot004,\ 0\cdot048.\\ R\sin\delta = 0\cdot017,\ 0\cdot042.\end{cases}$$

$$\nu_2\begin{cases}R\cos\delta = 0\cdot045,\ 0\cdot047.\\ R\sin\delta = -0\cdot124,\ -0\cdot112.\end{cases}$$

$$N_2\begin{cases}R\cos\delta = 0\cdot232,\ 0\cdot228.\\ R\sin\delta = 0\cdot582,\ 0\cdot578.\end{cases}$$

In general, the results for diurnal constituents are more irregular than those for semi-diurnal constituents. The computations for OO_1 were scrutinised without finding any error.

19. *Calculation of H and g.*

(*a*) Prepare forms with columns for all constituents, entering the constituents in the order of Table XXIV, and separating constituents of one species from those of another. Provision should be made for 10 rows of entries, as in the example, Table XXXV, and as in the annexed table.

	$2Q_1$	...
R cos δ R sin δ		
R f (T = 0) H = R/f		
δ Δ V (T = 0) u (T = 0) g = sum		

(*b*) At the top of each form enter the central day of the observations (T = 0) and the standard time (S) in which the observations are recorded. If the time-meridian is $s°$ west of Greenwich, enter $S = s/15$; but if it is $s°$ east of Greenwich, enter $S = -s/15$. Also enter the latitude and longitude of the place.

(*c*) For each constituent enter the values of R cos δ, R sin δ or the average values, if more than one have been evaluated.

(*d*) Enter the values of Δ from Table XXIV.

(*e*) Compute R, δ from the formulæ

$$R = +\sqrt{(R\cos\delta)^2 + (R\sin\delta)^2}$$
$$\tan\delta = R\sin\delta / R\cos\delta$$

and enter the results on the form.

Check the values of R and δ by computing R cos δ, R sin δ from them; a slide rule is very useful for this purpose.

(*f*) Compute the number of days elapsed from January 1 to the central day of the observations (T = 0), including leap day if one occurs.

(*g*) Compute s, h, p, N to two decimal places from the formulæ of Table XXV.

(*h*) Using the values of N and p, compute f, u from the formulæ of Table XXVI.

(*i*) Enter the values of V for each constituent computed from the formulæ of Table XXVII; for compound constituents use Table XXVIII, *e.g.*, the value of V for MP_1 is the V of M_2 minus the V of P_1.

(*j*) Enter the values of f, u; the value of u for compound constituents is computed like V; for f take the products of the values of f for the generating constituents; thus for MP_1 the value of f is $f(M_2) \times f(P_1)$.

(*k*) Compute H, g from the formulæ

$$H = R/f$$
$$g = \delta + \Delta + V + u.$$

(*l*) All computations must be independently checked.

20. *The Phase Lag.*

We have taken our time origin as zero hour of a particular day measured in standard time S hours later than Greenwich Mean Time. If V_P is the phase of a constituent of the tide-generating potential at the place at the precise moment of the time-origin, then, on substituting for the V used in the preceding paragraph, a quantity called κ would be obtained in place of g. It is clear that we have computed V at Greenwich S hours earlier than the precise time origin. Thus to get κ from g two corrections are necessary for Longitude and Time; if suffixes G and P denote quantities for Greenwich and the Place, then

$$V_P = V_G - pL + \sigma S,$$

whence

$$\kappa = g - pL + \sigma S,$$

where p is the suffix of the tidal constituent, L is the Longitude of the Place in degrees west of Greenwich, and σ is the phase-increment of the constituent in degrees per mean solar hour (Table I).

We have treated the observations in exactly the same way as observations taken at Greenwich, and the procedure is a convenient one as calculations involving L and S are eliminated both in analysis and prediction.

It may happen that the time-meridian for the place is changed from S to S_1 and the values of g must be amended to g'; we have

$$g = \kappa + pL - \sigma S, \qquad g' = \kappa + pL - \sigma S',$$

whence

$$g' - g = \sigma (S - S').$$

The formula is applicable if the observations have been taken in local mean time ($S = L/15$), and predictions are required in standard time S′ hours west of Greenwich.

The computer will readily understand that g must never be referred to as "kappa" and must not be quoted without S. Unfortunately, many sets of constants have been published which profess to give kappa but give some other form of phase lag, often loosely referred to as "kappa in standard time." This phrase should be avoided; it is frequently used for constants derived from observations treated as though the place was on the standard meridian.

TABLE XIV.—Hourly multipliers for X, Y.

	Hour.																							
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
X_0	1 .	. 1	1 1	. .	. 2	1 .	. 1	1 1	1 .	. 1	2 .	. .	1 1	1 .	. 1	2 .	1 .	1 .	2 .	. .	2 .	1 .	1 .	2 .
X_1	. . .	. -2 -2	. . .	. -1 -1	. . .	. -1 -1	. . .	. 1 -1	. . .	. 2 .	. . .	. 2 .	. . .	. 4 .	. . .	. 2 .	. . .	. 2 .	. . .	-1 1 .	. . .	-1 -1 .	. . .	-1 -1 .
Y_1	-1 1	. .	-1 -1	. .	-1 -1	. .	-2 -2	. .	-1 -1	. .	-1 -1	. .	1 -1	. .	2 .	. .	2 .	. .	4 .	. .	2 .	. .	2 .	. .
X_2	1 1	. .	2 2	. .	1 1	. .	-2 .	. .	-4 .	. .	-2 .	. .	2 .	. .	4 .	. .	2 .	. .	-2 .	. .	-4 .	. .	-2 .	. .
Y_2	. .	. -2	. .	1 1	. .	2 2	. .	1 1	. .	-2 .	. .	-4 .	. .	-2 .	. .	2 .	. .	4 .	. .	2 .	. .	-2 .	. .	-4 .
X_3	1 1 .	. 2 .	. 1 1	. . .	-1 -1 .	. -2 .	. -2 .	. . .	1 1 .	1 1 .	. 2 .	. . .	-2 . .	-1 -1 .	. -2 .	. . .	2 . .	1 1 .	1 1 .	. . .	-2 . .	-2 . .	-1 -1 .	. . .
X_4	. .	1 1	. .	. .	-1 -1	. .	. .	2 .	. .	. .	-2 .	. .	. .	2 .	. .	. .	-2 .	. .	. .	2 .	. .	. .	-2 .	. .
Y_4	. -2	. .	. .	1 1	. .	. .	-1 -1	. .	. .	2 .	. .	. .	-2 .	. .	. .	2 .	. .	. .	-2 .	. .	. .	2 .	. .	. .
X_6	1 2	. .	-1 -2	. .	2 1	. .	-2 -1	. .	3 .	. .	-3 .	. .	3 .	. .	-3 .	. .	3 .	. .	-3 .	. .	3 .	. .	-3 .	. .

The stencil hole for the date must appear on the same line as the symbol X, Y. Enter multipliers with negative sign in red ink.

TABLE XV.—Daily multipliers (d).

The full table extends from $T - \overline{T} = -14$ to 14. Multipliers for equal and opposite values of $T - \overline{T}$ have the same value and sign if the suffix is numeral but opposite signs if the suffix is literal.

$T-\overline{T}$.	Numeral suffix.								Literal suffix.						
	0	1	2	3	4	5	6	7	a	b	c	d	e	f	g
−14	1	−2	2	−2	2	−2	1	−1	0	−1	1	−1	1	−2	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0	1	2	2	2	2	2	2	2	0	0	0	0	0	0	0
1	1	2	2	2	1	1	1	0	−1	−1	−1	−2	−2	−2	−2
2	1	2	1	1	0	−1	−2	−2	−1	−1	−2	−2	−2	−1	0
3	1	1	1	−1	−2	−2	−2	0	−1	−2	−2	−1	0	1	2
4	1	1	0	−2	−2	−1	1	2	−1	−2	−1	1	2	2	1
5	1	1	−1	−2	−1	1	2	1	−2	−2	1	2	2	0	−2
6	1	1	−2	−1	1	2	1	−2	−2	−1	1	2	0	−2	−1
7	1	0	−2	−1	2	1	−2	−1	−2	−1	2	0	−2	−1	2
8	1	0	−2	1	2	−1	−1	2	−2	1	2	−1	−1	1	1
9	1	−1	−2	2	0	−2	1	1	−2	1	1	−2	1	2	−2
10	1	−1	−1	2	−1	0	2	−2	−2	2	0	−1	2	0	−1
11	1	−1	0	1	−2	1	0	−1	−1	2	−1	0	1	−2	1
12	1	−2	1	0	−2	2	−2	1	−1	2	−2	2	−1	−1	1
13	1	−2	2	−1	1	0	−1	1	−1	1	−2	1	−2	1	−1
14	1	−2	2	−2	2	−2	1	−1	0	1	−1	1	−1	2	−2

TABLE XVI.—Monthly multipliers (m).

Suffix.	$\overline{T}$.											
	−163.	−133.	−103.	−74.	−44.	−15.	15.	44.	74.	103.	133.	163.
0	1	1	1	1	1	1	1	1	1	1	1	1
1	−2	−1	−1	1	1	2	2	1	1	−1	−1	−2
2	1	0	−1	−1	0	1	1	0	−1	−1	0	1
3	−1	1	1	−1	−1	1	1	−1	−1	1	1	−1
a	−1	−2	−2	−2	−2	−1	1	2	2	2	2	1
b	1	2	1	−1	−2	−1	1	2	1	−1	−2	−1
c	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1

TABLE XVII.—Calculation of [R cos δ].

Principal function.	Correction terms :—multiples of							Divisor.	Constituent.
	X_{001}.	X_{002}.	X_{011}.	X_{020}.	X_{022}.				
X_{000}	−0·032	0·056	0·007	0·017	−0·003			10440	S_0
X_{001}	—	−0·136	0·050	—	0·005			9839	Sa
X_{002}	0·013	—	−0·003	—	−0·053			5688	Ssa
X_{011}	−0·012	+0·009	—	—	—			−9084	Mm
X_{0aa}	−0·207	0·007	—	—	0·015			−12917	Mm
X_{020}	—	−0·001	−0·013	—	0·005			10956	MSf
X_{022}	—	0·013	0·005	0·004	—			6571	Mf
X_{0bb}	—	0·302	0·021	0·043	—			9303	Mf
—	**Correction terms :—multiples of**								
	X_{301}.	X_{321}.	X_{3ba}.	X_{330}.	X_{341}.				
X_{301}	—	0·020	−0·030	—	—			9472	SK_3
X_{321}	—	—	—	—	0·020			10471	MK_3*
X_{3ba}	0·110	—	—	—	0·040			−14348	MK_3*
X_{330}	—	—	—	—	—			−11065	M_3
X_{341}	—	0·020	0·020	—	—			10689	MO_3
X_{3da}	0·050	0·050	—	—	—			11948	MO_3
	Correction terms :—multiples of								
	B_{40b}.	B_{4b0}.	B_{42b}.	B_{43a}.	B_{4d0}.	B_{45a}.			
A_{400}	−0·017	0·011	−0·005	—	0·012	0·001		5568	S_4
B_{40b}	—	0·004	0·104	—	0·003	—		9322	SK_4
B_{4b0}	0·006	—	0·062	−0·011	0·062	—		9724	MS_4
B_{42b}	0·013	−0·047	—	−0·011	0·005	0·001		9800	MK_4
B_{4b2}	−0·095	0·004	—	0·007	—	—		5292	MK_4
B_{43a}	—	—	—	—	—	0·007		11601	SN_4
B_{4c1}	0·007	—	0·026	—	—	0·088		−8097	SN_4
B_{4d0}	0·003	0·006	0·004	0·006	—	0·022		8584	M_4
B_{45a}	−0·004	—	−0·002	−0·055	—	—		9627	MN_4
B_{4e1}	0·004	—	0·016	0·074	—	—		−7991	MN_4
	Correction terms :—multiples of								
	X_{620}.	X_{622}.	X_{640}.	X_{642}.	X_{651}.	X_{660}.	X_{671}.		
X_{620}	—	0·120	0·090	—	—	0·090	−0·020	13087	$2SM_6$
X_{622}	—	—	—	0·240	—	—	0·010	6433	MSK_6
X_{6bb}	0·040	—	—	−0·270	—	—	—	−13693	MSK_6
X_{640}	0·020	—	—	0·200	0·010	0·050	—	12271	$2MS_6$
X_{642}	—	−0·020	0·010	—	−0·010	—	−0·010	5570	$2MK_6$
X_{6db}	—	0·150	0·080	—	0·020	—	—	−12283	$2MK_6$
X_{651}	—	0·020	—	0·020	—	—	0·040	−9848	MSN_6
X_{6ea}	—	—	—	0·030	—	—	0·120	−13657	MSN_6
X_{660}	0·080	—	−0·020	—	−0·030	—	−0·010	10845	M_6
X_{671}	—	—	—	0·010	0·080	—	—	−8778	$2MN_6$
X_{6ga}	—	0·010	—	0·020	0·020	—	—	−11550	$2MN_6$

TABLE XVIII.—Calculation of [R sin δ].

Principal function.	Correction terms :—multiples of							Divisor.	Constituent.
	X_{00a}.	X_{00b}.	X_{01a}.	X_{0b0}.	X_{02b}.				
X_{00a}	—	—0·037	0·050	—	—0·003			12665	Sa
X_{00b}	0·032	—	—0·005	0·003	—0·053			10117	Ssa
X_{01a}	—0·012	—0·001	—	—0·001	0·019			—13028	Mm
X_{0a1}	0·126	0·021	—	—	—0·002			9021	Mm
X_{0b0}	—0·002	0·005	0·016	—	—0·003			—10546	MSf
X_{02b}	—	0·012	0·008	—0·047	—			10518	Mf
X_{0b2}	—0·002	—0·095	—0·006	0·004	—			—5796	Mf

	Correction terms :—multiples of								
	X_{30a}.	X_{32a}.	X_{3b1}.	X_{3c0}.	X_{34a}.				
X_{30a}	—	0·020	0·050	—	—			12198	SK_3
X_{32a}	—	—	—	—	0·020			14123	MK_3*
X_{3b1}	—0·060	—	—	—	—0·030			10637	MK_3*
X_{3c0}	—	—	—	—	—			—10994	M_3
X_{34a}	—	0·020	—0·030	—	—			—13014	MO_3
X_{3d1}	—0·030	—0·030	0·010	—	—			9811	MO_3

	Correction terms :—multiples of								
	B_{402}.	B_{420}.	B_{422}.	B_{431}.	B_{440}.	B_{451}.			
B_{400}	0·052	0·017	—0·005	—	0·019	—		—9644	S_4
B_{402}	—	—	0·103	—	—	0·001		—5238	SK_4
B_{420}	—	—	0·117	0·020	0·088	—0·002		—10055	MS_4
B_{422}	0·013	0·004	—	—0·013	—0·001	0·002		—4878	MK_4
B_{4bb}	0·302	0·043	—	—0·013	—	—		10579	MK_4
B_{431}	—	—	0·016	—	—0·004	0·007		8559	SN_4
B_{4ca}	—0·006	—	0·007	—	—	—0·148		10988	SN_4
B_{440}	—	0·016	0·004	—0·006	—	0·012		—9273	M_4
B_{451}	—	0·001	0·022	—0·055	—0·001	—		7430	MN_4
B_{4ea}	—0·003	—	0·005	—0·135	0·001	—		10350	MN_4

	Correction terms :—multiples of								
	X_{6b0}.	X_{62b}.	X_{6d0}.	X_{64b}.	X_{65a}.	X_{6f0}.	X_{67a}.		
X_{6b0}	—	0·060	0·060	0·010	—	—	—	12749	$2SM_6$
X_{62b}	—0·050	—	—0·020	0·240	—	0·010	—	12918	MSK_6
X_{6b2}	—	—	—	0·050	—	—	—	6822	MSK_6
X_{6d0}	—	—	—	0·080	0·020	0·020	—	11461	$2MS_6$
X_{64b}	—	—0·020	—0·100	—	—0·010	—	—	12827	$2MK_6$
X_{6d2}	—	—0·040	0·010	—	—0·020	—	—	5369	$2MK_6$
X_{65a}	—	—	—	—	—	—	0·040	12752	MSN_6
X_{6e1}	—	0·020	—	0·030	—	—0·010	—0·070	—10538	MSN_6
X_{6f0}	0·010	—	0·040	0·010	0·020	—	—0·020	10640	M_6
X_{67a}	—	—	—	—	0·080	—	—	11012	$2MN_6$
X_{6g1}	—	0·020	—	0·020	—0·010	—	—	—9207	$2MN_6$

TABLE XIX.—Calculation of [R cos δ].

Principal function.	Correction terms :—multiples of							Divisor.	Principal constituent.
	A_{101}.	B_{10a}.	A_{110}.	A_{112}.	A_{121}.	A_{123}.	A_{132}.		
A_{100}	—0·033	—	—0·018	—0·018	—	0·002	—0·001	10452	S_1
A_{101}	—	—	—	0·044	—0·020	—0·003	+0·001	9782	K_1*
B_{10a}	—	—	—	0·023	0·017	—0·021	—0·007	9700	K_1*
A_{102}	0·012	—	—	0·113	—	—0·004	0·017	5775	π_1*
B_{10b}	—	0·032	—	0·165	—	—0·011	—0·049	—7732	π_1*
A_{103}	—0·006	—	—	—0·023	0·002	—0·086	—0·001	6376	ϕ_1
B_{10c}	—	—0·012	—	—0·023	0·002	—0·078	0·011	5219	ϕ_1
A_{110}	—	—	—	0·125	0·003	—0·005	—0·007	—10658	M_1*
B_{1a0}	—	0·004	—	—0·093	0·001	0·001	0·011	—8242	M_1*
A_{112}	—	—	0·001	—	—0·005	0·015	—0·110	—5309	J_1*
B_{11b}	—	—	—0·017	—	—0·001	0·040	0·115	—7764	J_1*
B_{1a2}	—	—0·001	0·001	—	—0·001	—0·006	—0·011	3949	J_1*
A_{1ab}	0·007	—	0·024	—	—0·001	0·023	—0·028	—8766	J_1*
A_{121}	0·004	—	—	—0·057	—	—0·054	0·058	11785	O_1**
B_{12a}	—	0·004	—	—0·038	—	—0·249	—0·024	—10305	O_1**
B_{1b1}	—	—0·065	—	0·075	—	0·040	0·012	7569	O_1**
A_{1ba}	0·107	—	—0·001	—0·071	—	—0·248	0·072	13401	O_1**
A_{123}	—	—	—	0·029	—0·029	—	—0·057	6115	OO_1
B_{12c}	—	—	—	0·027	—0·028	—	0·022	5893	OO_1
B_{1b3}	—	—	—	—0·035	—0·019	—	—0·020	—4311	OO_1
A_{1bc}	0·001	—	—0·001	0·054	0·036	—	—0·029	5849	OO_1
A_{130}	—	—	—0·053	—0·005	—0·003	+0·006	+0·053	—11608	ρ_1
B_{1c0}	—	0·001	0·054	0·025	—0·002	—0·007	0·030	—7649	ρ_1
A_{132}	—	—	—0·002	—0·017	0·003	—0·009	—	—7135	Q_1
B_{13b}	—	—	0·006	—0·024	0·001	—0·045	—	7639	Q_1
B_{1c2}	—	—	—0·001	—0·153	0·003	0·016	—	—4186	Q_1
A_{1cb}	0·002	—	—0·011	0·338	—	—0·054	—	—10675	Q_1
A_{141}	0·002	—	—	0·006	0·039	0·003	—0·027	11885	σ_1
B_{14a}	—	0·002	—	0·006	—0·035	0·043	0·028	—9153	σ_1
B_{1d1}	—	—0·029	—	0·008	0·032	—0·007	—0·021	6897	σ_1
A_{1da}	0·049	—	—	—0·008	0·057	0·010	—0·055	13278	σ_1
A_{143}	—	—	—	—0·001	—0·002	0·094	0·024	6510	$2Q_1$
B_{14c}	—	—	—0·001	—0·001	—0·002	0·092	—0·018	—5743	$2Q_1$
B_{1d3}	—	—	—	—0·009	0·004	—0·114	0·029	3648	$2Q_1$
A_{1dc}	0·001	—	0·001	0·011	—0·006	0·154	0·015	8517	$2Q_1$

TABLE XX.—Calculation of [R sin δ].

Principal function.	Correction terms :—multiples of							Divisor.	Principal constituent.
	B_{101}.	A_{10a}.	B_{110}.	B_{1ab}.	B_{1ba}.	B_{1bc}.	B_{1cb}.		
B_{100}	—0·033	—	—0·019	—0·010	—	0·002	—0·002	—8020	S_1
B_{101}	—	—	0·001	0·026	—0·017	—0·002	—0·003	—7547	K_1*
A_{10a}	—	—	—	—0·018	—0·029	0·039	0·009	12572	K_1*
B_{102}	0·013	—	0·001	0·072	—	—0·006	0·019	—4571	π_1*
A_{10b}	—0·001	0·032	—0·004	—0·141	—0·001	0·011	0·051	—9886	π_1*
B_{103}	—0·006	—	—	—0·014	—0·003	—0·091	—0·004	—5053	ϕ_1
A_{10c}	—	—0·012	—	0·023	0·005	0·106	—0·008	6844	ϕ_1
B_{110}	—	—	—	0·076	0·003	—0·005	0·003	7796	M_1*
A_{1a0}	—	0·004	—	0·085	—0·002	—0·002	—0·004	—11263	M_1*
B_{112}	—	—	0·002	—	—0·003	0·020	—0·075	4408	J_1*
A_{11b}	—0·001	—	0·033	—	—0·002	—0·046	—0·181	—9322	J_1*
A_{1a2}	—	—0·002	—0·001	—	0·003	0·009	0·017	4858	J_1*
B_{1ab}	0·007	—	0·024	—	—	0·024	—0·018	7110	J_1*
B_{121}	0·003	—	—0·001	—0·028	—	—0·058	0·035	—8237	O_1**
A_{12a}	0·001	0·004	—0·001	0·056	—	0·356	0·068	—14574	O_1**
A_{1b1}	—	—0·064	—0·001	—0·078	—	—0·056	—0·032	10827	O_1**
B_{1ba}	0·106	—	—0·002	—0·039	—	—0·261	0·040	—9473	O_1**
B_{123}	—0·003	—	—	0·009	—0·025	—	—0·036	—5172	OO_1
A_{12c}	0·005	—	—0·001	—0·035	0·050	—	—0·038	6824	OO_1
A_{1b3}	0·004	—	0·001	0·040	0·033	—	0·036	—5118	OO_1
B_{1bc}	0·005	—	—	0·029	0·032	—	—0·016	—4993	OO_1
B_{130}	—	—	—0·053	—0·005	—0·002	—0·006	0·036	7701	ρ_1
A_{1c0}	—	+0·001	—0·102	—0·036	+0·004	+0·013	—0·050	—11571	ρ_1
B_{132}	—	—	—0·002	—0·010	0·002	—0·009	—	4735	Q_1
A_{13b}	—	—	—0·012	0·024	0·002	0·073	—	11502	Q_1
A_{1c2}	—	—0·001	0·006	0·141	—0·005	—0·026	—	—6484	Q_1
B_{1cb}	0·003	—	—0·006	0·205	—0·002	—0·055	—	6925	Q_1
B_{141}	0·005	—	—	—	0·034	—0·003	—0·018	—7515	σ_1
A_{14a}	0·006	0·002	—	—0·021	0·058	—0·107	—0·045	—14464	σ_1
A_{1d1}	—0·006	—	—	—	—0·058	0·020	0·031	10900	σ_1
B_{1da}	0·054	—	—	—0·015	0·051	—0·004	—0·035	—8393	σ_1
B_{143}	—	—	—	0·005	0·004	0·097	0·018	—4057	$2Q_1$
A_{14c}	—0·001	—	0·001	+0·004	—0·006	—0·133	0·023	—9226	$2Q_1$
A_{1d3}	—	—	—	0·002	0·004	0·166	—0·038	5897	$2Q_1$
B_{1dc}	0·001	—	0·001	0·013	0·003	0·159	0·014	—5298	$2Q_1$

TABLE XXI.—Calculation of [R cos δ].

Principal function.	Correction terms :—multiples of									Divisor.	Principal constituent.
	A_{201}.	A_{202}.	A_{211}.	B_{21a}.	A_{220}.	A_{231}.	B_{23a}.	A_{240}.	A_{242}.		
A_{200}	—0·032	0·056	—0·004	—0·008	0·017	—0·004	—0·004	0·019	0·002	11812	S_2
A_{201}	—	—0·136	—0·023	—0·052	—	—0·021	—0·019	—	—0·005	11154	T_2*
B_{20a}	—	—0·065	—0·108	—0·028	—	—0·036	—0·026	—	0·010	—14300	T_2*
A_{202}	0·013	—	—0·001	0·001	—	—0·001	—0·001	—	—0·016	6406	K_2
B_{20b}	—0·041	—	0·011	0·004	0·001	0·004	0·003	0·002	0·024	11443	K_2
A_{211}	—0·012	—0·006	—	—	—	0·004	0·043	—0·003	—0·005	—9486	L_2**
B_{21a}	0·016	—0·003	—	—	—	0·082	0·016	0·001	0·006	—14185	L_2**
B_{2a1}	—0·160	0·037	—	—	—	0·019	0·012	—0·002	—0·012	—10662	L_2**
A_{2aa}	—0·207	0·007	—	—	—	—0·029	—0·028	—	—0·018	16674	L_2**
A_{213}	—	0·003	—0·022	0·015	—	—0·002	—	—0·002	0·003	—6986	KJ_2
B_{21c}	—	0·005	—0·020	—0·060	—	—	0·002	0·009	—0·003	—8308	KJ_2
B_{2a3}	—	—0·020	—0·020	0·014	—	—	—0·002	—	0·010	6099	KJ_2
A_{2ac}	—0·001	0·028	0·030	0·065	—	0·002	—	—0·005	0·010	—7096	KJ_2
A_{220}	—	—	—0·003	0·007	—	0·003	—0·013	0·088	0·011	13040	M_2*
B_{2b0}	0·002	0·009	—0·004	0·013	—	0·007	—0·007	0·051	0·003	11911	M_2*
A_{222}	—	0·013	0·003	—0·002	0·004	—0·010	0·003	—0·001	—0·027	7879	OP_2*
B_{22b}	—	0·022	—0·010	0·002	—0·043	0·018	0·002	0·005	0·040	—11879	OP_2*
B_{2b2}	—0·003	—0·170	0·003	—0·004	0·003	—0·006	0·003	—	0·017	6416	OP_2*
A_{2bb}	—	0·302	0·020	—0·001	0·043	—0·010	0·002	—0·001	0·022	11109	OP_2*
A_{231}	—0·001	—	—0·052	—0·011	—	—	—	—0·004	0·016	—11405	N_2*
B_{23a}	0·001	—	—0·022	—0·053	—	—	—	0·001	—0·020	14249	N_2*
B_{2c1}	—0·049	0·010	0·069	—0·038	—	—	—	—	—0·030	—10109	N_2*
A_{2ca}	—0·063	0·002	0·084	—0·071	—	—	—	—	0·045	—14869	N_2*
A_{240}	—	—	0·007	0·001	0·016	0·005	0·009	—	0·090	12694	μ_2
B_{2d0}	0·001	0·004	0·006	0·004	0·005	0·004	0·009	—	0·074	10561	μ_2
A_{242}	—	0·005	—0·002	—	—	0·004	—	0·014	—	7039	$2N_2$
B_{24b}	—	0·009	—0·002	—0·005	—	—0·011	—0·006	—0·081	—	—10251	$2N_2$
B_{2d2}	—0·001	—0·078	—0·001	—0·001	—	0·004	—0·001	0·012	—	5756	$2N_2$
A_{2db}	—	0·138	0·007	0·005	0·001	0·014	0·006	0·083	—	10487	$2N_2$
A_{251}	—	—	—0·021	—0·005	—	—0·067	—0·010	—0·001	—0·060	—10475	MNS_2
B_{25a}	—	—	—0·010	—0·022	—	—0·019	—0·070	0·001	0·070	11854	MNS_2
B_{2e1}	—0·027	0·005	0·070	—0·020	—	—0·130	—0·026	—	—0·050	—9918	MNS_2
A_{2ea}	—0·035	—	0·041	—0·074	—	0·057	0·159	—0·001	—0·074	—14696	MNS_2
A_{253}	—	—	—	—	—	—	0·003	—	0·045	—6483	OQ_2
B_{25c}	—	—	—0·001	—0·001	—0·004	—0·006	—0·004	0·002	—0·038	7798	OQ_2
B_{2e3}	—	—0·003	0·002	—0·002	—	—	0·006	—	0·037	—6008	OQ_2
A_{2ec}	—	0·005	—	—0·004	0·002	0·013	0·011	0·003	0·040	—9557	OQ_2

TABLE XXII.—Calculation of [R sin δ].

Principal function.	Correction terms :—multiples of									Divisor.	Principal constituent.
	B_{201}.	A_{20b}.	B_{211}.	A_{21a}.	B_{220}.	B_{231}.	A_{23a}.	B_{240}.	B_{2db}.		
B_{200}	—0·034	—0·032	—0·004	0·007	0·017	—0·004	0·003	0·020	0·003	—11812	S_2
B_{201}	—	0·077	—0·024	0·050	—	—0·022	0·017	—	—0·006	—11074	T_2*
A_{20a}	—	—0·036	0·115	—0·028	—	0·042	—0·026	—0·001	—0·007	—14357	T_2*
B_{202}	0·013	—	—0·001	—0·001	—	—0·001	0·001	—0·001	—0·001	— 6444	K_2
A_{20b}	0·042	—	—0·012	0·003	—0·001	—0·005	0·003	—0·003	—0·020	11377	K_2
B_{211}	—0·012	0·003	—	—	—	0·004	—0·037	—0·003	—0·004	9253	L_2**
A_{21a}	—0·015	—0·001	—	—	—	—0·097	0·016	—0·001	—0·005	—14549	L_2**
A_{2a1}	0·164	0·020	—	—	—	—0·022	0·012	0·003	0·010	—10929	L_2**
B_{2aa}	—0·207	—0·003	—	—	—	—0·029	0·023	—0·001	—0·012	—16263	L_2**
B_{213}	—	—0·002	—0·021	—0·014	—	—0·002	0·001	—0·001	0·003	7213	KJ_2
A_{21c}	—	0·002	0·023	—0·059	—	—	0·002	—0·011	0·002	— 8045	KJ_2
A_{2a3}	—0·001	—0·010	0·022	0·014	—	—	—0·001	—	—0·008	5900	KJ_2
B_{2ac}	—0·003	—0·016	0·031	—0·062	—	0·003	—	—0·004	0·007	7309	KJ_2
B_{220}	—	—	—0·003	—0·007	—	0·003	0·011	0·088	0·007	—12365	M_2*
A_{2b0}	—0·003	0·005	0·004	0·013	—	—0·009	—0·006	—0·063	—0·003	12560	M_2*
B_{222}	—0·001	—0·006	0·003	0·002	0·003	—0·009	—0·003	—0·002	—0·018	— 7437	OP_2*
A_{22b}	—	0·015	0·010	0·001	0·046	—0·021	0·003	—0·009	—0·034	—12617	OP_2*
A_{2b2}	—0·004	—0·094	—0·005	—0·004	—0·004	0·007	0·003	—	—0·010	6926	OP_2*
B_{2bb}	—0·009	—0·171	0·022	0·001	0·042	—0·009	—0·002	0·001	0·021	—10246	OP_2*
B_{231}	—0·001	—	—0·052	0·010	—	—	—	—0·003	0·011	10511	N_2*
A_{23a}	—0·001	—0·001	0·023	—0·053	—	—	—	—	0·017	15460	N_2*
A_{2c1}	0·048	0·005	—0·073	—0·038	—	—	—	—0·002	—0·025	—10962	N_2*
B_{2ca}	—0·063	—0·003	0·084	0·066	—	—	—	0·002	0·030	13705	N_2*
B_{240}	—	—0·004	0·007	—0·001	0·016	0·006	—0·008	—	0·060	—11367	μ_2
A_{2d0}	—0·001	0·007	—0·007	0·004	—0·006	—0·006	0·010	—	—0·062	11704	μ_2
B_{242}	—	—0·003	—0·002	—	—	0·004	—	0·014	—	— 6301	$2N_2$
A_{24b}	—	0·005	0·002	—0·005	0·001	0·014	—0·006	0·100	—	—11450	$2N_2$
A_{2d2}	—0·001	—0·043	0·001	—0·001	—	—0·005	—0·001	—0·015	—	6446	$2N_2$
B_{2db}	—0·004	—0·078	0·008	—0·005	—0·001	0·015	—0·006	0·083	—	— 9363	$2N_2$
B_{251}	—0·001	0·003	—0·022	0·005	—	—0·067	0·009	—0·004	—0·040	9160	MNS_2
A_{25a}	0·002	0·005	0·011	—0·022	—	0·022	—0·069	—0·005	—0·060	13586	MNS_2
A_{2e1}	—0·035	0·001	—0·076	—0·019	—	0·152	—0·026	0·002	0·042	—11357	MNS_2
B_{2ea}	—0·034	0·006	0·040	0·071	—	0·056	—0·137	—0·004	—0·051	12865	MNS_2
B_{253}	—	—0·002	—	—	—	—	—0·003	0·002	0·031	5652	OQ_2
A_{25c}	—	—0·002	0·001	—0·002	0·004	0·007	—0·005	—0·001	0·032	8981	OQ_2
A_{2e3}	—	0·001	—0·002	—0·001	—	—0·001	0·006	—0·003	—0·031	— 6923	OQ_2
B_{2ec}	—	—0·005	—	0·004	0·004	0·013	—0·010	0·004	0·027	8323	OQ_2

TABLE XXIII.—Calculation of R cos δ from [R cos δ] and R sin δ from [R sin δ].

A, B, C, D are the four values of [R cos δ] or [R sin δ] obtained by the use of Tables XVII to XXII.

Principal constituent.	Constituent.	R cos δ. Multiples of A.	B.	C.	D.	R sin δ. Multiples of A.	B.	C.	D.
K_1*	K_1	0·498	0·502	—	—	0·502	0·498	—	—
	P_1	0·497	—0·497	—	—	0·497	—0·497	—	—
π_1*	π_1	0·50	0·50	—	—	0·50	0·50	—	—
	ψ_1	0·50	—0·50	—	—	0·50	—0·50	—	—
M_1*	M_1	0·52	0·48	—	—	0·47	0·53	—	—
	θ_1	0·55	—0·55	—	—	0·55	—0·55	—	—
J_1*	J_1	0·55	—	0·45	—	0·60	—	0·40	—
	J_1	—	0·60	—	0·40	—	0·55	—	0·45
	χ_1	0·51	—	—0·51	—	0·50	—	—0·50	—
	χ_1	—	0·45	—	—0·45	—	0·45	—	—0·45
O_1**	O_1	0·28	0·22	0·24	0·26	0·24	0·26	0·28	0·22
	MP_1	0·24	—0·24	0·24	—0·24	0·24	—0·24	0·24	—0·24
	SO_1	0·30	—0·30	—0·27	0·27	0·30	—0·30	—0·27	0·27
T_2*	T_2	0·50	0·50	—	—	0·50	0·50	—	—
	R_2	0·50	—0·50	—	—	0·50	—0·50	—	—
L_2**	L_2	0·22	0·24	0·26	0·28	0·22	0·24	0·26	0·28
	λ_2	0·465	—0·465	—	—	0·465	—0·465	—	—
	λ_2	—	—	0·54	—0·54	—	—	0·54	—0·54
	MSN_2	0·25	0·27	—0·25	—0·27	0·25	0·27	—0·25	—0·27
M_2*	M_2	0·53	0·47	—	—	0·47	0·53	—	—
	$2SM_2$	0·52	—0·52	—	—	0·52	—0·52	—	—
OP_2*	OP_2	0·55	0·45	—	—	0·55	0·45	—	—
	OP_2	—	—	0·55	0·45	—	—	0·55	0·45
	MKS_2	0·55	—0·55	—	—	0·55	—0·55	—	—
	MKS_2	—	—	0·55	—0·55	—	—	0·55	—0·55
N_2*	N_2	0·55	0·45	—	—	0·55	0·45	—	—
	N_2	—	—	0·55	0·45	—	—	0·55	0·45
	ν_2	0·515	—0·515	—	—	0·515	—0·515	—	—
	ν_2	—	—	0·48	—0·48	—	—	0·48	—0·48
MK_3	MK_3	0·45	0·55	—	—	0·50	0·50	—	—
	SO_3	0·51	—0·51	—	—	0·51	—0·51	—	—

TABLE XXIV.—Values of Δ.

—	Δ.	—	Δ.	—	Δ.	—	Δ.
	°		°		°		°
A_0	—	$2Q_1$	199·24	OQ_2	63·80	MN_4	80·07
Sa	0·78	σ_1	200·37	MNS_2	65·07	M_4	88·51
Ssa	1·56	Q_1	207·68	$2N_2$	72·38	SN_4	95·82
Mm	10·34	ρ_1	208·81	μ_2	73·51	MS_4	104·25
MSf	19·30	O_1	216·12	N_2	80·82	MK_4	105·53
Mf	20·86	MP_1	217·39	ν_2	81·94	S_4	120·00
		M_1	224·63	OP_2	87·98	SK_4	121·27
		χ_1	225·83	M_2	89·25		
MO_3	353·18	π_1	231·23	MKS_2	90·53		
M_3	6·90	P_1	231·86	λ_2	96·56	$2MN_6$	306·12
SO_3	18·58	S_1	232·50	L_2	97·69	M_6	314·28
MK_3	20·63	K_1	233·14	T_2	104·36	MSN_6	321·36
SK_3	46·03	ψ_1	233·77	S_2	105·00	$2MS_6$	329·52
		ϕ_1	234·41	R_2	105·64	$2MK_6$	330·76
		θ_1	240·45	K_2	106·27	$2SM_6$	344·76
		J_1	241·57	MSN_2	113·44	MSK_6	345·99
		SO_1	248·88	KJ_2	114·71		
		OO_1	250·16	$2SM_2$	120·75		

TABLE XXV.—Values of *s*, *h*, *p*, N.

s, *h*, *p*, N are the mean longitudes of the moon, sun, moon's perigee, and moon's ascending node respectively.

Y = the year.
D = the number of days elapsed since January 1 in the year Y.
l = the integral part of $\frac{1}{4}$(Y − 1901) = the number of leap years between 1900 and Y, excluding Y as the leap day in this year is counted in D.

$$\left.\begin{aligned} s &= 277^\circ{\cdot}025 + 129^\circ{\cdot}38481\,(Y-1900) + 13^\circ{\cdot}17640\,(D+l) \\ h &= 280{\cdot}190 - 0{\cdot}23872\,(Y-1900) + 0{\cdot}98565\,(D+l) \\ p &= 334{\cdot}385 + 40{\cdot}66249\,(Y-1900) + 0{\cdot}11140\,(D+l) \\ N &= 259{\cdot}157 - 19{\cdot}32818\,(Y-1900) - 0{\cdot}05295\,(D+l) \end{aligned}\right\}\text{at zero hour of day D, G.M.T.}$$

TABLE XXVI.—Values of *f* and *u*.

	f: series of multiples of				*u*: series of multiples of		
	1.	cos N.	cos 2N.	cos 3N.	sin N.	sin 2N.	sin 3N.
					°	°	
Mm	1·0000	−0·1300	0·0013	—	0·00	0·00	0·00
Mf	1·0429	0·4135	−0·0040	—	−23·74	2·68	−0·38
O_1	1·0089	0·1871	−0·0147	0·0014	10·80	−1·34	0·19
K_1	1·0060	0·1150	−0·0088	0·0006	−8·86	0·68	−0·07
J_1	1·0129	0·1676	−0·0170	0·0016	−12·94	1·34	−0·19
OO_1	1·1027	0·6504	0·0317	−0·0014	−36·68	4·02	−0·57
M_2	1·0004	−0·0373	0·0002	—	−2·14	—	—
K_2	1·0241	0·2863	0·0083	−0·0015	−17·74	0·68	−0·04
L_2	$f\cos u = 1 - 0{\cdot}2505\cos 2p - 0{\cdot}1102\cos(2p-N) - 0{\cdot}0156\cos(2p-2N) - 0{\cdot}0370\cos N.$ $f\sin u = -0{\cdot}2505\sin 2p - 0{\cdot}1102\sin(2p-N) - 0{\cdot}0156\sin(2p-2N) - 0{\cdot}0370\sin N.$						
M_1	$f\cos u = 2\cos p + 0{\cdot}4\cos(p-N).$ $f\sin u = \sin p + 0{\cdot}2\sin(p-N).$						

TABLE XXVII.—V, f and u.

V is given only for zero hour of the mean solar day at Greenwich. For compound constituents the entries under s, h, p are omitted (see Table XXVIII). A dash under f, u indicates that $f = 1$, $u = 0$ invariably.

	V.				f, u.		V.				f, u.		V.				f, u.
	s	h	p	°			s	h	p	°			s	h	p	°	
Sa	0	1	0		—	$2Q_1$	−4	1	2	270	O_1	OQ_2					
Ssa	0	2	0		—	σ_1	−4	3	0	270	O_1	MNS_2					
M*m*	1	0	−1		M*m*	Q_1	−3	1	1	270	O_1	$2N_2$	−4	2	2		M_2
MS*f*						ρ_1	−3	3	−1	270	O_1	μ_2	−4	4	0		M_2
M*f*	2	0	0		M*f*	O_1	−2	1	0	270	O_1	N_2	−3	2	1		M_2
						MP_1						ν_2	−3	4	−1		M_2
						M_1	−1	1	0	90	M_1	OP_2					
						χ_1	−1	3	−1	90	J_1	M_2	−2	2	0		M_2
M_3	−3	3	0	180	*	π_1	0	−2	0	192	—	MKS_2					
						P_1	0	−1	0	270	—	λ_2	−1	0	1	180	M_2
						S_1	0	0	0	180	—	L_2	−1	2	−1	180	L_2
						K_1	0	1	0	90	K_1	T_2	0	−1	0	282	—
						ψ_1	0	2	0	168	—	S_2	0	0	0		—
						ϕ_1	0	3	0	90	—	R_2	0	1	0	258	—
						θ_1	1	−1	1	90	J_1	K_2	0	2	0		K_2
						J_1	1	1	−1	90	J_1	MSN_2					
						SO_1						KJ_2					
						OO_1	2	1	0	90	OO_1	$2SM_2$					

* For M_3
$f = f^{\frac{3}{2}}(M_2)$
$u = \frac{3}{2}u(M_2)$

(In this table the arguments for ψ_1, T_2, R_2 involve p_1, which has been taken $= 282°$, its value about the year 1950.)

TABLE XXVIII.—Relations between the values of V, u, or $V + u$ for compound constituents and those for the generating constituents, applicable only at zero hour of mean solar day at Greenwich.

MS*f*	$-M_2$	OQ_2	O_1+Q_1	MO_3	M_2+O_1	MN_4	M_2+N_2	$2MN_6$	M_4+N_2
		MNS_2	M_2+N_2	M_3	Tab. XXVII	M_4	M_2+M_2	M_6	M_4+M_2
		OP_2	O_1+P_1	SO_3	O_1	SN_4	N_2	MSN_6	M_2+N_2
		MKS_2	M_2+K_2	MK_3	M_2+K_1	MS_4	M_2	$2MS_6$	M_4
MP_1	M_2	MSN_2	M_2-N_2	SK_4	K_1	MK_4	M_2+K_2	$2MK_6$	M_4+K_2
SO_1	$-O_1$	KJ_2	K_1+J_1			S_4	—	$2SM_6$	M_2
		$2SM_2$	$-M_2$			SK_4	K_2	MSK_6	M_2+K_2

The value of f for a compound constituent is the product of the values of f for the generating constituents.

TABLE XXIX.—Illustrating use of stencil for calculating X_2 from hourly heights of tide.

Tide-Gauge record at VANCOUVER, B.C., April, 1921.

Time used : Pacific Standard, S = 8.

Date	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Apl. 1	10·6	11·1	10·9	10·3	9·5	8·7	8·0	7·7	7·7	8·0	8·5	8·8	8·8	8·3	7·6	6·6	5·4	4·4	3·7	3·5	4·2	5·3	6·8	8·3
,, 2	9·8	11·0	11·5	11·4	10·6	9·8	8·8	8·0	7·6	7·5	7·9	8·4	8·8	9·0	8·8	8·0	7·1	6·0	4·9	4·1	3·9	4·5	5·6	7·0
,, 3	8·6	10·0	11·0	11·3	10·9	10·0	8·9	7·9	7·0	6·4	6·4	7·0	7·8	8·5	8·9	8·8	8·1	7·1	5·9	4·8	3·9	3·7	4·4	5·7
,, 4	7·1	8·7	10·0	10·7	10·7	10·0	9·0	7·6	6·4	5·5	5·2	5·5	6·4	7·4	8·4	8·9	8·9	8·1	7·0	5·7	4·6	3·8	3·7	4·5
	1		2		1		−2		−4		−2		2		4		2		−2		−4		−2	
7	5·0 5·0		8·1 7·5		10·8 10·4		10·5		7·6		4·2		2·9		5·4		9·2		10·8		8·8		6·0	
	1		2		1																			
													Stencil for X_2.											
,, 11	7·8	6·9	6·9	7·8	9·0	10·4	11·4	11·7	10·8	9·4	7·5	5·3	3·2	1·6	0·7	1·5	3·2	5·4	7·8	9·9	11·4	12·0	11·6	10·5
,, 11	9·3	8·1	7·5	7·8	8·8	9·9	11·1	11·9	11·8	10·8	9·3	7·5	5·5	3·7	2·3	1·8	2·8	4·6	6·8	9·0	10·9	12·2	12·5	12·0
,, 13	10·8	9·6	8·7	8·1	8·3	9·2	10·2	11·1	11·5	11·2	10·0	8·4	6·7	4·6	2·8	1·7	1·5	2·6	4·5	6·6	8·7	10·5	11·9	12·3
…																								
…																								

TABLE XXX.—Example of daily process, $X_2 + 500$. (Vancouver, 1920–1, T = 0 on April 1, 1921.)

	Date and day number (T) of first entry in each column :—											
	Oct. 6	Nov. 4.	Dec. 4.	Jan. 3.	Feb. 1.	Mar. 3.	Apr. 1.	May 1.	May 30	June 29.	July 28.	Aug. 27.
$T-\bar{T}$.	−177	−148	−118	−88	−59	−29	0	30	59	89	118	148
		661	672		657		643		630		618	630
−14	660	677	673	663	656	653	652	646	647	635	643	657
−13	687	674	662	661	647	653	652	655	658	653	660	669
−12	673	658	640	645	631	643	643	655	657	660	664	662
−11	652	630	614	620	608	627	627	644	647	658	651	636
−10	621	598	586	596	583	606	606	627	622	636	622	595
− 9	582	568	560	565	561	582	580	601	591	607	583	548
− 8	554	552	539	551	541	560	556	573	556	569	540	514
− 6	534	534	528	533	531	538	533	544	528	532	507	495
− 7	519	527	528	525	520	523	517	523	509	507	493	498
− 5	520	529	530	519	516	513	514	510	510	500	505	521
− 4	530	539	542	529	527	517	525	515	530	515	531	554
− 3	552	559	560	541	545	534	550	540	564	548	570	589
− 2	575	579	586	564	572	558	584	575	604	587	608	620
− 1	600	607	606	592	604	592	623	614	640	625	638	643
0	625	631	640	620	637	624	656	650	665	653	655	654
1	645	654	657	647	661	658	674	673	670	664	657	653
2	658	663	668	664	671	673	672	674	664	659	647	642
3	661	663	663	668	657	666	651	659	637	643	632	627
4	654	650	639	651	630	642	616	630	608	622	611	607
5	635	625	609	618	593	603	580	597	581	595	587	583
6	607	589	564	574	548	562	548	568	557	572	563	561
7	572	551	528	532	514	529	526	545	541	553	545	542
8	543	525	508	503	495	507	517	530	530	537	530	527
9	518	503	502	490	498	505	518	525	528	528	522	518
10	507	506	516	503	519	516	533	530	534	526	522	523
11	518	537	550	533	549	541	553	543	546	533	531	536
12	540	568	591	574	585	567	579	563	564	546	548	560
13	577	612	630	607	617	598	606	585	586	566	573	593
14	623	645	656	639	638	620	628	609	611	591	603	630

TABLE XXXI.—Example of monthly process (Vancouver, 1920–21).

$\bar{T}$ =	−163	−133	−103	−74	−44	−15	15	44	74	103	133	163
X_{20} + 5000	4742	4753	4675	4527	4454	4510	4619	4707	4685	4620	4541	4557
X_{21} + 300	266	239	217	214	260	225	282	318	368	388	389	262
X_{2a} + 300	212	217	200	236	234	279	321	337	415	393	385	289
X_{22} + 10000	11352	11734	12001	11732	11954	11587	11839	11448	11717	11350	11725	12070
X_{2b} + 10000	8066	8432	8769	8398	8754	8374	8727	8348	8644	8269	8516	8784
X_{23} + 2000	1743	1581	1628	1582	1863	1933	2272	2346	2493	2531	2402	2065
X_{2c} + 2000	2297	1968	1720	1705	1563	1624	1741	1824	2123	2150	2412	2515
X_{24} + 1000	1014	1027	952	930	956	936	1035	1050	1023	1020	918	891
X_{2d} + 1000	1005	1036	1028	1052	985	946	944	945	1009	1051	1079	1013

TABLE XXXII.—Example of annual process (Vancouver, 1920–21).

Suffix.	X.	Y.	Suffix.	X.	Y.	Suffix.	X.	Y.
200	—4614	4405	220	20509	—17111	2*c*1	—3929	—1157
201	—560	30	2*b*0	—17919	—20805	2*ca*	3441	—6023
20*a*	204	—203	222	48	—177	240	—248	333
202	—79	723	22*b*	—824	—939	2*do*	93	198
20*b*	1429	247	2*b*2	—129	—40	242	— 49	—361
211	—115	—225	2*bb*	—599	466	24*b*	653	—109
21*a*	1119	1119	231	926	—4027	2*d*2	—232	57
2*a*1	225	806	23*a*	6877	4265	2*db*	—242	—1049
2*aa*	1405	—1249						

TABLE XXXIII.—Example of alternative method (Vancouver, 1920–21).

$T-\bar{T}$.	$X_{2.0}+7000$.	$X_{2.1}+2000$.	$X_{2.a}+2000$.	$X_{2.2}+1000$.	$X_{2.b}+1000$.
—14	7662	1960	1800	1004	1072
—13	7731	1870	1945	1027	1067
—12	7631	1868	2113	1019	1039
—11	7414	1898	2240	1003	1029
—10	7098	1978	2262	988	1042
— 9	6728	2064	2220	969	1061
— 8	6405	2117	2066	969	1099
— 7	6137	2119	1926	979	1105
— 6	5989	2068	1837	988	1094
— 5	5987	1963	1864	1009	1057
— 4	6154	1890	1940	1010	1004
— 3	6452	1839	2087	1012	982
— 2	6812	1849	2217	996	968
— 1	7184	1918	2290	995	975
0	7510	1995	2251	981	1013
1	7713	2087	2114	992	1042
2	7755	2126	1939	990	1062
3	7627	2078	1791	994	1074
4	7360	1991	1729	999	1073
5	7006	1903	1755	998	1090
6	6613	1843	1910	1011	1099
7	6278	1837	2085	1015	1084
8	6052	1866	2186	1016	1084
9	5955	1960	2233	1011	1041
10	6035	2054	2169	1000	1012
11	6270	2100	1998	986	1024
12	6585	2125	1838	971	1023
13	6950	2082	1712	985	1049
14	7293	1992	1687	1004	1064

TABLE XXXIV.—Example of calculation of [R cos δ] and R cos δ (Vancouver, 1920–21).

Principal term.		Correction terms arising from A_{201}	A_{202}	A_{211}	B_{21a}	A_{220}	A_{231}	B_{23a}	A_{240}	A_{242}	Sum.	Divisor.	[R cos δ]	Princ. Const.	R cos δ.	Const.
		—530	644	—340	0	3398	—3101	2612	85	—410						
A_{200}	—209	17	36	1	0	58	12	—10	2	— 1	— 98	11812	—0·008	S_2	—0·008	S_2
A_{201}	—530	—	—87	8	0	—	65	—50	—	2	—592	11154	—0·053	T_2*	—0·042	T_2
B_{20a}	407	—	—42	37	0	—	112	—68	—	— 4	442	—14300	—0·031	T_2*	—0·011	R_2
A_{202}	644	— 7	—	0	0	—	3	— 3	—	7	644	6406	0·101	K_2	0·101	K_2
B_{20b}	1182	22	—	— 4	0	3	— 12	8	0	—10	1189	11443	0·104	K_2	0·104	K_2
A_{211}	—340	6	— 4	—	—	—	— 12	112	0	2	—236	— 9486	0·025	L_2**	0·026	L_2
B_{21a}	0	— 9	— 2	—	—	—	—254	42	0	— 2	—225	—14185	0·016	L_2**	0·004	λ_2
B_{2a1}	—581	85	24	—	—	—	— 56	31	0	5	—492	—10657	0·046	L_2**	0·015	λ_2
A_{2aa}	156	111	4	—	—	—	90	—71	0	7	298	16678	0·018	L_2**	—0·006	MSN_2
A_{220}	3398	—	—	1	0	—	— 9	—34	7	— 5	3358	13040	0·257	M_2*	0·248	M_2
B_{2b0}	2886	—1	6	1	0	—	— 22	—18	4	— 1	2855	11911	0·239	M_2*	0·009	$2SM_2$
...															...	...
...															...	...

TABLE XXXV.—Example of calculation of H and g (Vancouver, 1920–21).

T = 0 on April 1, 1921. Y — 1900 = 21. D = 90. l = 5.
S = 8. Lat. = 49° 18′ N. Long. = 123° 07′ W.
s = 285°·864. h = 8°·814. p = 118°·880. N = 208°·235.

——	$2SM_2$.	K_2.	—	L_2.	λ_2.	M_2.	—
R cos δ	0·009	0·102		0·026	0·010	0·248	
R sin δ	0·017	0·135		—0·112	0·045	—3·070	
R	0·019	0·169		0·115	0·046	3·077	
f (T = 0)	1·033	0·777		1·000	1·033	1·0334	
H = R/f	0·018	0·218		0·105	0·044	2·979	
δ	64·8	52·9		283·0	77·5	274·6	
Δ	120·75	106·27		97·69	96·56	89·25	
V (T = 0)	194·10	17·63		152·88	13·02	165·90	
u (T = 0)	—1·01	8·98		9·17	1·01	1·01	
g = sum	18·6	185·8		182·7	188·1	170·8	